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HIGH-ACCURACY POSITIONING OF LATHE SERVO SYSTEM USING FUZZY CONTROLLERS BASED ON VARIABLE UNIVERSE OF DISCOURSE

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Abstract: A novel fuzzy controller is proposed here to improve the positioning precision of servo system of CNC lathes. To utilize the high–accuracy potential of the VUD fuzzy controller, a grating–ruler was used as a sensor to measure the displacement of work pieces. Designing the VUD fuzzy controller involved five steps, i.e., setting up universes of discourse and parameters, selecting membership functions, designing a differential circuit, constructing a base of fuzzy rules, and defining a set of contraction-expansion factors. The method of lookup table was applied to construct the base of fuzzy rules via four typical input instructions, and the method of maximum strength to settle the conflicting rules properly. The experimental results show that the VUD fuzzy controller can be effective in controlling the position of work pieces.

Index terms*:* servo system, positioning of work pieces, CNC-lathe, fuzzy controller, fuzzy system, variable universe of discourse, contraction–expansion factor, approximation error.

I. INTRODUCTION

Servo feed systems form the key parts of computer numerical control (CNC) lathes, because they play an important role in positioning and tracking work pieces. It is known that machining precision, surface smoothness and production rate depend partly on the accuracy of servo systems. To obtain high precision, PID controllers are often applied, because they do not need mathematical model during the designing process. Besides, they are simple in principle, easy to implement, and have a wide scope of application [1, 2, 3]. However, their parameters need a field tuning, which continues to be a challenge to most engineers. It is known that the effect of PID controlling depends on the quality of parameter configuration; improper configuration may lead to bad performance or even failure in controlling, necessitating thereby the development of a different controller. Generally, fuzzy controllers are preferred to PID controllers, because they are robust and do not need field configuration for parameters. But many application examples show that most fuzzy controllers cannot satisfy the requirement of high accuracy systems, because they lack integral elements. So, in recent decades, many scholars and engineers are engaged in improving the output accuracy of fuzzy controllers.

Variable universe of discourse (VUD) is one of effective methods to improve control accuracy. In VUD fuzzy controllers, the universes of discourse can be adjusted online by a set of contraction–expansion factors. The VUD method improves the control accuracy significantly, although the number of fuzzy rules needed is reduced sharply. Earlier, nonlinear modification of universe of discourse was proposed in [4]. Li *et al.* introduced systematically the theory of VUD fuzzy systems [5, 6]. In 2002, they conducted a successful experiment on a quadruple inverted pendulum using VUD fuzzy controllers [7, 8]. Their achievements were widely received, because controlling quadruple inverted pendulum was a great challenge at that time. Many researchers developed a keen interest in the theoretical analysis and practical design of VUD fuzzy controllers. For example, Wang *et al.* discussed some VUD methods on the chaotic systems and auto gauge control system [9–11]; Shan *et al.* developed an Analog Circuit with VUD fuzzy systems [12]; Long *et al.* investigated the approximation properties of fuzzy systems and their design method [13–16]. All these research works are some examples of how to obtain a good performance in the control accuracy.

To construct a VUD fuzzy controller, a complete set of fuzzy rules is needed, and there are two methods to obtain it. One is to refine the experience of experts or technicians, and the other is to use the method of lookup table. This paper focuses on the latter, because it is more suitable for designing the controller of CNC lathes. In addition, to maximize the advantage of fuzzy controller in output precision, a grating–rule sensor was used.

The remainder of the paper is organized as follows. After introduction, in Section II, the basic structure of the control system of CNC lathes is presented; Section III discusses about optical grating detector; Section IV presents the steps involved in the design of fuzzy controllers; Section V discusses the experimental results of VUD fuzzy controllers. Finally, Section VI presents the conclusions drawn from this study.

II. THE CONTROL GOAL AND BASIC CONSTRUCTION OF VUD FUZZY CONTROL SYSTEM

A servo feed system of a lathe does not satisfy the requirement on the X–axis positioning accuracy. The feed movement on the X–axis was driven by a DC torque motor, and the position feedback data of work piece collected by a photo–electricity encoder. The model number of DC torque motor was J255LYX02; its peak voltage at locked–rotor was 48V, peak current at locked–rotor 7.5A, peak torque at locked–rotor 23 N.M, continuous voltage at locked–rotor 24V, continuous current at locked–rotor 3.75A, and continuous torque at locked–rotor 11.5 N.M. The structure of servo control system is shown in Figure 1. The positioning accuracy of X–axis feed system is 10.0μm. The present task is to improve the accuracy to the level of 2.0μm.

Based on the requirement on servo feed controlling, the positioning accuracy of servo feed system can be improved in two ways, that is by upgrading the detection precision of sensors or by using a new control scheme. Both the methods should be applied to achieve the best effect. For this study, a grating–rule sensor was used to improve the detection accuracy and a VUD method followed to overcome the shortage of general fuzzy controllers.

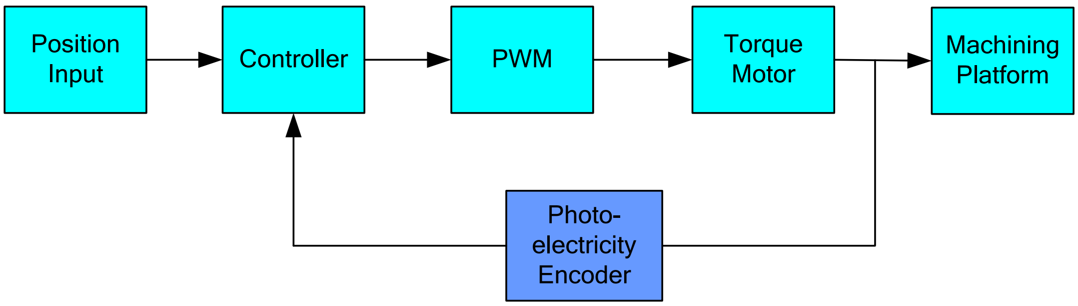

Figure 1. The structure of servo feed system based on a Photo–electricity Encoder

III. GRATING–RULE SENSORS

The photo–electricity encoder is a sensor used for measuring angular displacement. It can measure displacement with high–accuracy, but not linear displacement of work pieces. Angular displacement can be transformed into linear displacement by driving of ball screw, but the transformation introduces an error which should be taken into account. So, the data obtained from the photo–electricity encoder will be inaccurate and hence cannot represent the real position of work piece. For effective improvement of the positioning accuracy of work piece, the first step is to select a suitable sensor for the control system. The grating–ruler shown in Figure 2 is considered the right sensor for measuring linear displacement in the servo feed system, and hence the same was used for this study.

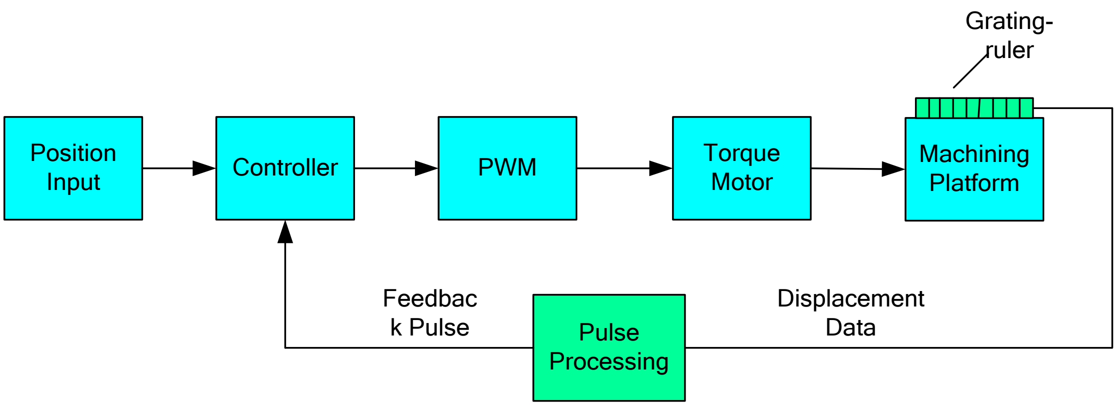

Figure 2. The experimental set-up diagram of servo feed system based on a grating ruler

The grating ruler includes two phototransistors whose output is near sinusoidal signal corresponding to a moiré fringe of grating. The output is driven by a circuit with differential amplifying, reshaping, subdividing, and directional identifying. It is finally counted by a reversible counter. The displacement of machining platform is measured by photo–electrical detecting circuit in conjunction with the moiré fringe from the grating shift. The experimental set-up diagram of grating measurement is shown in Figure 3.

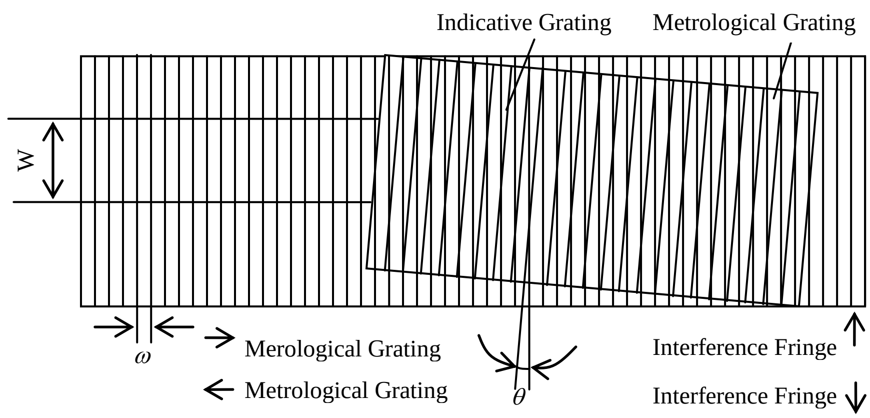

Figure 3. The schematic of grating measurement

The grating is usually categorized into two types: 1) Transmission grating; 2) Reflection grating. Transmission grating is done with glass materials which easily break in the movement of work pieces; so, for this study, reflection grating was chosen. Reflection grating consists generally of stainless sheet steel on which 100 pairs/mm pierced gaps and light–proof stripes are carved. The grating ruler is generally installed by the following methods:

a) The indicative grating is installed on the guide rail and the metrological grating on the machine stand. To avoid friction, the gap between these gratings is maintained in the range of 0.03~0.06 mm.
b) A small intersection angle is needed so that moiré fringes are generated on indicative grating when the light beam passes through the grating.

When metrological grating is stationary and indicative grating moves in horizontal direction, moiré fringes shift in the vertical direction. Moiré fringes are applied to amplify signals and homogenize errors. The direction and number of moiré fringes are controlled by that of grating grooves. Moiré fringes are collected by photo–electricity sensors. The alternating dim and bright fringes convey the periodicity of the voltage signals collected by photo–electricity sensors. The periodical voltage signals can be converted into count pulses by the electronic technique of amplifying, transforming and reshaping. The number of pulses indicates the number of shifted grating grooves. The physical displacement of grating grooves can be calculated by

$$W = \omega / (2\sin\theta / 2) \approx \omega / \theta \approx k_w \omega \,, \tag{1}$$

where ω represents the grid distance which indicates the number of grating grooves per mm, W the width of moiré fringes, and k_w the magnification factor of grating. Because the number of shifted grating grooves determines that of moiré fringes, as also that of count pulses, the physical displacement of grating can be easily obtained via (1).

Subdivision technology can be used to improve the resolution capability of phototransistors without added grating grooves. When two gratings are one grid apart, the period of output signals of phototransistors is 2π. Thus, if the output signals are counted directly, the resolution of phototransistors is one grid only. To identify a displacement smaller than one grid, a frequency multiplier of 16 times is considered. In the frequency multiplier, 16 count pulses are partitioned equally in the grating gap. That is to say, the frequency of count pulses rises by 16 times. If the grating gap is 0.01mm, the phototransistors can identify the displacement of 0.625μm by the subdivision technology. Thus, the measurement accuracy of the displacement of machining platform reaches micrometer grade. Thus, this forms a good hardware platform for high accuracy in piece positioning.

IV. THE DESIGN OF VUD FUZZY CONTROLLERS

a. Structure of VUD fuzzy control systems

To construct controllers and analyze their performance, mathematical models based on transform functions are usually applied. So, it is necessary to establish the transform functions for PWM power amplifier, motor, and grating sensor. These functions can be obtained by mathematical derivation. As the difference between the real system and the derived model is considerable in some cases that use the circuit theory, electrical machine principles and dynamics knowledge, it is better to consider the least squares identification algorithm based on input–output data. By identification experiments, the transform function of the PWM amplifier is obtained as

$$G_p(s) = \frac{30}{0.0008s+1}. \tag{2}$$

The transform function of the torque motor is obtained as

$$G_m(s) = \frac{0.26}{(0.0015s+1)(0.017s+1)} \tag{3}$$

The transform function of the grating sensor is obtained as

$$G_g(s) = \frac{1}{0.006s+1} \tag{4}$$

For effective positioning accuracy, a double–input–single–output VUD fuzzy controller serves as the core of the servo system. The closed loop system is shown in Figure 4.

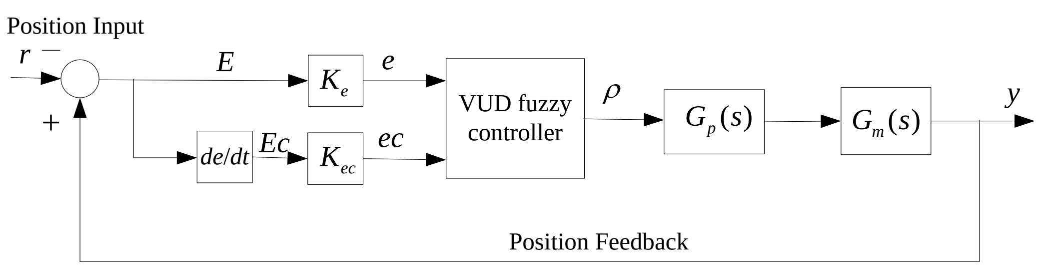

Figure 4. The close loop system based on a VUD fuzzy controller

In Figure 4, r represents the input of control system, E the position error, Ec the rate of error change, e the normalized position error, ec the normalized rate of error change, and ρ the output of controller (PWM duty ratio).

b. Steps in designing VUD fuzzy controllers

For the closed loop system shown in Figure 4, VUD fuzzy controller can be constructed by the following steps:

Step 1: to set up universes of discourse and parameters. Let the universe of discourse for e be $[-1\text{mm}, 1\text{mm}]$, for ec $[-1\,\text{mm} / \text{s}, 1\,\text{m}$, and for ρ $[-1, 1]$, in which $[-1, 0]$ represents the duty cycle of backward shift and $[0, 1]$ the duty cycle of forward shift. By testing the original servo system, the error E reaches a maximum of 190 mm, and that of the rate of error change Ec 25.87mm/s. Thus, one obtains

$$K_e = \frac{1}{190} = 0.00526\,, \tag{5}$$

$$K_{ec} = \frac{1}{25.87} = 0.03865\,, \tag{6}$$

Denoting the duty cycle of PWM as ρ, one gets

$$\rho = \frac{t_{on}}{T} \tag{7}$$

where T represents the switching cycle of PWM, t_{on} the switch–on time, and U_s the voltage of DC input power. The average input voltage of torque motor can be denoted by $U_m = \rho U_s$.

Step 2: to select membership functions. The triangle–shaped functions are selected as the membership functions of fuzzy sets, because the requirements of VUD fuzzy controller are low on the shape of membership functions.

Step 3: to design a differential circuit. Use a 2nd order differential circuit to connect with the input–output terminals e 、ec 、u of the original servo controller, as shown in Figure 5.

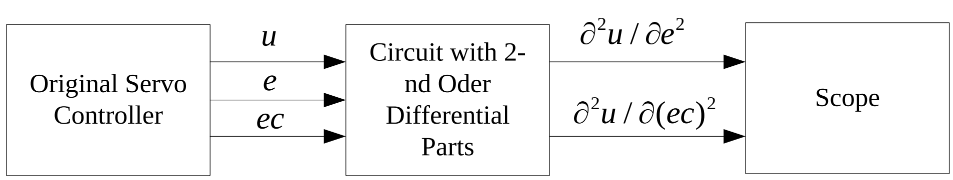

Figure 5. The schematic diagram of circuit with 2nd order differential parts

By giving the typical signal of input position, a scope is used to view the signals output $\partial^2 u / \partial e^2$ and $\partial^2 u / \partial (ec)^2$. Thus, one gets

$$\left\| \partial^2 u / \partial e^2 \right\|_\infty = 1.96 \tag{8}$$

$$\left\| \partial^2 u / \partial (ec)^2 \right\|_\infty = 1.35 \tag{9}$$

Suppose that the approximation accuracy ε is 0.05 and that all universes of discourse are partitioned equally by the same number of intervals. Then, the universes of discourse can be partitioned by

$$M = \mathrm{INT}\left(U \sqrt{\sum_{i=1}^{n} \left\| \frac{\partial^2 u}{\partial x_i^2} \right\|_\infty \Big/ 8\varepsilon} \right) + 1. \tag{10}$$

where M represents the number of fuzzy sets on the corresponding universe of discourse, U the positive boundary value of the corresponding universe of discourse, and INT a function that rounds off its independent variable to the nearest integer value toward minus infinity.

From Step 1, it is known that U is set to be 1. Using (8), (9), and (10), one gets $M = 6$. That is to say, six fuzzy sets are partitioned equally on the universes of discourse of e and ec. Their initial membership functions are shown in Figures 6 and 7, where *NB*, *NM*, *NS*, *PS*, *PM* and *PB* represent negative big, negative medium, negative small, positive small, positive medium and positive big, respectively. The central points of the six fuzzy sets are -1, -0.67, -0.33, 0.33, 0.67, and 1. The universe of discourse of output ρ is partitioned as shown in Figure 8.

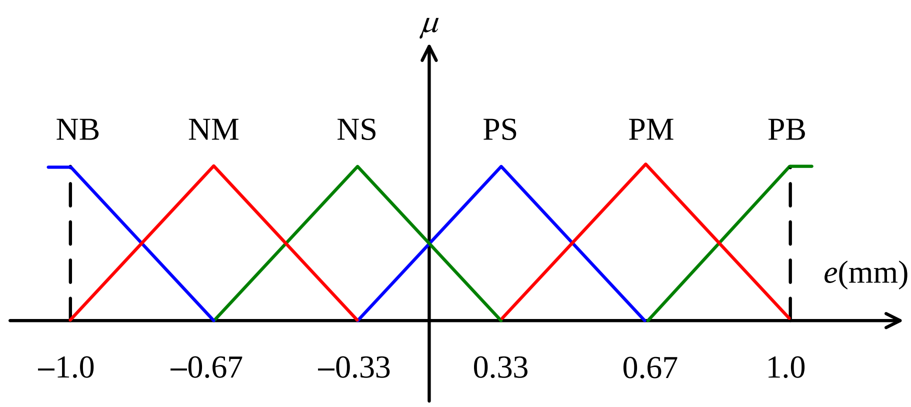

Figure 6. The initial membership functions of e

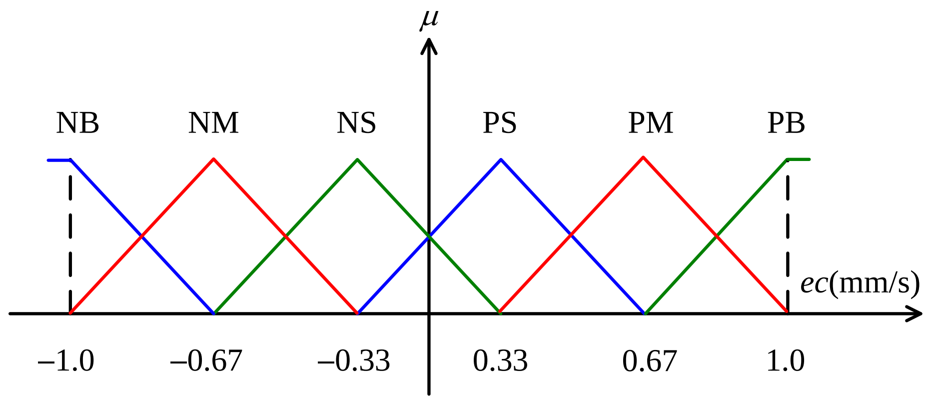

Figure 7. The initial membership functions of ec

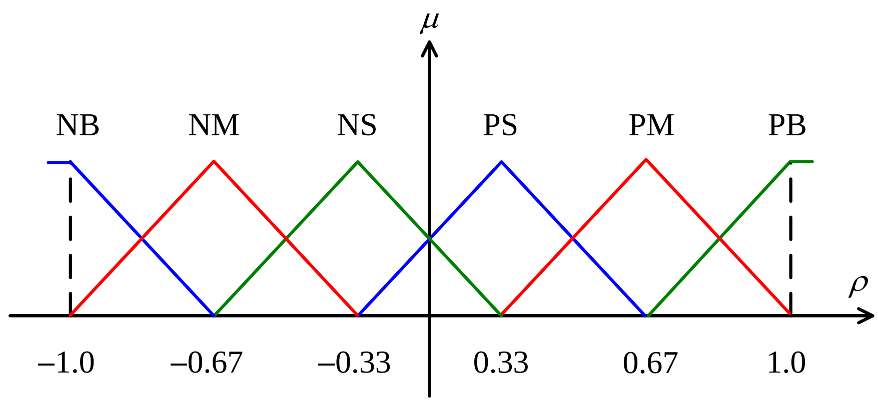

Figure 8. The initial membership functions of ρ

Denoting the sum of fuzzy rules as $M^{'}$, one gets $M^{'}=\left(M+1\right)^{n}=(5+1)^{2}=36$. Denote the fuzzy rules thus:

$$\text{If } e \text{ is } A_{1r_1} \text{ and } ec \text{ is } A_{2r_2}\text{, then } \rho \text{ is } B_{r_o}\text{,} \tag{11}$$

where A_{1r_1} is a fuzzy set corresponding to error e, $r_1=1,2,\cdots,6$, and A_{2r_2} corresponding to the rate of error change ec, $r_2=1,2,\cdots,6$. Thus, one has $A_{11}=NB$, $A_{12}=NM$, $A_{13}=NS$, $A_{14}=PS$, $A_{15}=PM$, $A_{16}=PB$, $A_{21}=NB$, $A_{23}=NS$, $A_{24}=PS$, $A_{25}=PM$, $A_{26}=PB$.

Step 4: to construct a base of fuzzy rules using the method of lookup table. At first, set the work piece at the middle of rail, start the original servo system and input four typical position instructions (-190mm, -50mm, +50mm, +190mm) one by one, and measure the input–output data $(e^k, ec^k; \rho^k)$ of servo system at the adjusting phase by the sample period of 0.1 second. The data $(e^k, ec^k; \rho^k)$ is shown in Table. 1

Table. 1 The input-output data at the typical positions

k	$(e^k, ec^k; \rho^k)$			
	$r = -190$	$r = -50$	$r = 50$	$r = 190$
0	(-1.000,0.000;0.000)	(-0.264,0.000;0.000)	(0.264,0.000;-0.000)	(1.000,0.000;0.000)
1	(-0.932,0.497;1.000)	(-0.208,0.423;0.343)	(0.211,0.420;-0.345)	(0.932,0.497;-1.000)
2	(-0.801,0.965;0.921)	(-0.152,0.412;0.332)	(0.149,0.419;-0.330)	(0.808,0.959;-0.925)
3	(-0.670,0.956;0.735)	(-0.119,0.253;0.257)	(0.110,0.259;-0.249)	(0.672,0.952;-0.738)
4	(-0.554,0.857;0.721)	(-0.084,0.255;0.156)	(0.080,0.251;-0.159)	(0.561,0.855;-0.724)
5	(-0.448,0.773;0.702)	(-0.060,0.179;0.127)	(0.060,0.174;-0.128)	(0.451,0.775;-0.706)
6	(-0.352,0.699;0.662)	(-0.038,0.161;0.084)	(0.036,0.164;-0.082)	(0.348,0.695;-0.670)
7	(-0.289,0.463;0.509)	(-0.025,0.096;0.073)	(0.021,0.099;-0.071)	(0.292,0.465;-0.511)
8	(-0.224,0.470;0.326)	(-0.014,0.081;0.062)	(0.010,0.082;-0.061)	(0.217,0.475;-0.318)
9	(-0.168,0.412;0.287)	(-0.008,0.044;0.036)	(0.005,0.034;-0.032)	(0.161,0.414;-0.282)
10	(-0.118,0.368;0.234)	(-0.005,0.022;0.031)	(0.002,0.021;-0.029)	(0.122,0.370;-0.233)
11	(-0.075,0.316;0.143)	(-0.002,0.021;0.022)	(0.001,0.009;-0.012)	(0.069,0.326;-0.131)
12	(-0.056,0.142;0.112)	(-0.001,0.007;0.013)	(0.000,0.002;0.000)	(0.051,0.141;-0.107)
13	(-0.038,0.132;0.086)	(0.000,0.001;0.000)	(0.000,0.001;0.000)	(0.035,0.124;-0.088)
14	(-0.023,0.113;0.070)	(0.000,0.000;0.000)	(0.000,0.000;0.000)	(0.021,0.110;-0.068)
15	(-0.011,0.086;0.052)	(0.000,0.001;0.000)	(0.000,0.001;0.000)	(0.010,0.081;-0.054)
16	(-0.006,0.038;0.033)	(0.000,0.000;0.000)	(0.000,0.000;0.000)	(0.005,0.035;-0.032)
17	(-0.002,0.031;0.021)	(0.000,0.001;0.000)	(0.000,0.001;0.000)	(0.002,0.027;-0.018)
18	(-0.001,0.007;0.009)	(0.000,0.000;0.000)	(0.000,0.000;0.000)	(0.001,0.006;-0.008)
19	(0.000,0.001;0.000)	(0.000,0.000;0.000)	(0.000,0.001;0.000)	(0.000,0.000;0.000)
20	(0.000,0.000;0.000)	(0.000,0.001;0.000)	(0.000,0.000;0.000)	(0.000,0.000;0.000)

Based on the data corresponding to $r = -190$, the method of constructing fuzzy rules will be as follows. For the case of $k=1$, Table 1 indicates that $(e^1, ec^1; \rho^1) = \left(-0.932, 0.497; 1.000\right)$. From Figure 6, one can calculate the degree by which e^1 belongs to the membership function *NB*, *NM*, and other fuzzy sets as 0.794, 0.206, and 0, respectively. Similarly, one can calculate from Figure 7 the degree by which ec^1 belongs to the membership function *PM*, *PS*, and other fuzzy sets as 0.509, 0.491, and 0, respectively. From Figure 8, one can calculate the degree by which ρ^1 belongs to the membership function *PB* and other fuzzy sets as 1 and 0, respectively. The fuzzy set with the maximum value of membership function was selected as the desired fuzzy set. Thus, e^1 belongs to the membership function *NB*, ec^1 to *PM*, and ρ^1 to *PB*. Hence, a fuzzy rule should be written as follows:

$$\text{If } e \text{ is } NB \text{ and } ec \text{ is } PM \text{, then } \rho \text{ is } PB. \tag{12}$$

The strength D is defined as follows:

$$D = \mu_{A_{1r1}}(e)\mu_{A_{2r2}}(ec)\mu_{B_{ro}}(\rho), \tag{13}$$

where A_{1r1}, A_{2r2}, B_{r_o} are consistent with those given in (11). Then, the strength D corresponding to the fuzzy rule (12) can be obtained as follows:

$$D_{r=-190,k=1} = 0.794 \times 0.509 \times 1 = 0.404 \tag{14}$$

For the case of $k=2$, Table 1 indicates that $(e^2, ec^2; \rho^2) = \left(-0.801, 0.965; 0.921\right)$. Using the same method as the one used for the case of $k=1$, one can obtain the degree by which e^2 belongs to the membership function *NB*, *NM*, and other fuzzy sets as 0.397, 0.603, and 0, respectively. Similarly, one can obtain from Figure 7 the degree by which ec^2 belongs to the membership function *PB*, *PM*, and other fuzzy sets as 0.894, 0.106, and 0, respectively. Also, from Figure 8, one can determine the degree by which ρ^2 belongs to the membership function *PB*, *PM* and other fuzzy sets as 0.761, 0.239 and 0, respectively. Thus, e^2 belongs to the membership function *NM*, ec^2 to *PB*, and ρ^2 to *PB*. Hence, another fuzzy rule should be written as follows:

$$\text{If } e \text{ is } NM \text{ and } ec \text{ is } PB \text{, then } \rho \text{ is } PB. \tag{15}$$

Then, the strength D corresponding to the fuzzy rule (12) is obtained as follows:

$$D_{r=-190,k=2} = 0.603 \times 0.894 \times 0.761 = 0.410 \tag{16}$$

Similarly, for the other 18 pairs of data under the condition of $r = -190$, 18 other rules were obtained. For the other three conditions of $r = -50$, $r = 50$, $r = 190$, 60 pairs of data were obtained (see Table 1). Using this data, 60 other rules were obtained. Thus, the sum of rules totals up to 80. For a set of rules with the same antecedent IF and different consequent THEN, their strength D was used to pick out the rule that holds the maximum strength D and to delete the other rules. Then, the conflicting rules were properly settled. For easy application of rules, fuzzy sets *NB*, *NM*, *NS*, *PS*, *PM* and *PB*, corresponding to ρ were de–fuzzified to -1, -0.67, -0.33, 0.33, 0.67, 1. Thus, a complete base of fuzzy rules was obtained and the same is presented in Table. 2.

Table. 2 Fuzzy rules of the duty cycle ρ

e	*ec*					
	NB	*NM*	*NS*	*PS*	*PM*	*PB*
NB	1.000	1.000	1.000	1.000	0.677	0.677
NM	1.000	1.000	1.000	0.677	0.677	0.677
NS	0.677	0.677	0.677	0.333	0.333	0.333
PS	-0.333	-0.333	-0.333	-0.677	-0.677	-0.677
PM	-0.677	-0.677	-0.677	-1.000	-1.000	-1.000
PB	-0.677	-0.677	-1.000	-1.000	-1.000	-1.000

Step 5: to define a set of contraction-expansion factors. Based on the results in [13], the factors $\alpha_e(e^k)$, $\alpha_{ec}(e^k)$ and $\beta(\rho^k)$ are defined as follows.

$$\alpha_e(e^k) = \begin{cases} 1, & k = 0, \\ \left|e^k\right|^{\tau} + 0.0001, & k = 1,\ 2,\ 3,\ \cdots, \end{cases} \tag{17}$$

$$\alpha_{ec}(e^k) = \begin{cases} 1, & k = 0, \\ \left|ec^k\right|^{\tau} + 0.0001, & k = 1,\ 2,\ 3,\ \cdots, \end{cases} \tag{18}$$

$$\beta(\rho^k) = \begin{cases} 1, & k = 0, \\ \left|\rho^k\right|^{\tau} + 0.0001, & k = 1,\ 2,\ 3,\ \cdots, \end{cases} \tag{19}$$

where the parameter τ is set at 0.9.

It is already known that the expression of VUD fuzzy controllers can be written as follows.

$$y^{k+1} = f(\boldsymbol{x}^k) = \beta(y^k) \frac{\sum_{j=1}^{m} \bar{y}_j^0 \prod_{i=1}^{n} \mu_{A_{ij}^0}\left(\frac{x_i^k}{\alpha_i(x_i^k)}\right)}{\sum_{j=1}^{m} \prod_{i=1}^{n} \mu_{A_{ij}^0}\left(\frac{x_i^k}{\alpha_i(x_i^k)}\right)} \tag{20}$$

where $\bar{y}_j$ is the central point of B_j. When x_1^k, x_2^k and $\bar{y}_j$ are substituted by e^k, ec^k and ρ, respectively, the VUD fuzzy controllers can be rewritten as follows:

$$\rho^{k+1} = \beta(\rho^k) \frac{\sum_{j=1}^{36} \bar{\rho}_j^0 \mu_{A_{1j}^0}\left(\frac{e^k}{\alpha_e(e^k)}\right) \mu_{A_{2j}^0}\left(\frac{ec^k}{\alpha_{ec}(ec^k)}\right)}{\sum_{j=1}^{36} \mu_{A_{1j}^0}\left(\frac{e^k}{\alpha_e(e^k)}\right) \mu_{A_{2j}^0}\left(\frac{ec^k}{\alpha_{ec}(ec^k)}\right)}, \tag{21}$$

where ρ^{k+1} and ρ^k represent the duty cycles of $(k+1)$th and kth sample steps, respectively, and $\bar{\rho}_j^0 (j = 1, 2, \cdots, 6)$ represents the central points of output fuzzy sets. The sample period is set at 0.001s.

V. EXPERIMENTAL RESULTS

When the authors input a position testing instruction (-50mm), a set of data: $e^k = -0.25$, $ec^k = 0.12$ and $\rho^k = 0.722$ are collected via the output data ports of VUD fuzzy controller, where $k = 600$. Using (17) and (18),

$$\alpha_e(e^k) = \left|e^k\right|^{\tau} + 0.0001 = 0.25^{0.9} + 0.0001 = 0.2873,$$

and

$$\alpha_{ec}(e^k) = \left|ec^k\right|^{\tau} + 0.0001 = 0.12^{0.9} + 0.0001 = 0.1485.$$

Thus, the universes of discourse of e and ec are shrunk to be

$$U_e = [-0.2873, 0.2873]$$

$$U_{ec} = [-0.1485, 0.1485]$$

From Figure 6, one can deduce that the central points of fuzzy sets *NB, NM, NS, PS, PM, PB* in the universe of discourse U_e are shifted to -0.2873，-0.1925，-0.0948，0.0948，0.1925，0.2873. Similarly, the central points of fuzzy sets *NB, NM, NS, PS, PM, PB* in the universe of discourse U_{ec} are shifted to −0.1485，−0.0995，−0.0490，0.0490，0.0995，0.1485. Thus, for the sample step k, the fuzzy sets of e and ec can be drawn as Figures 9 and 10.

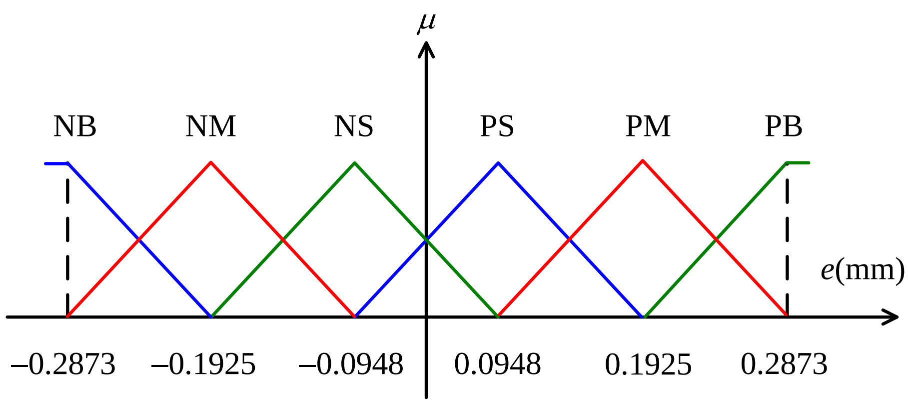

Figure 9. The membership functions of e^k at the sample step k

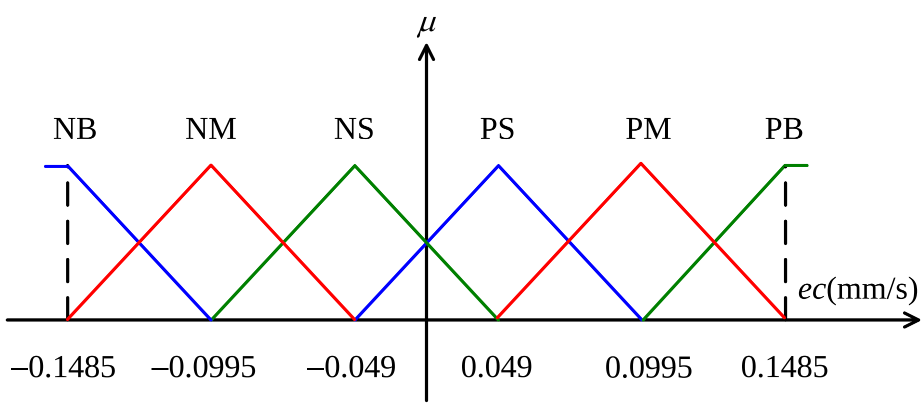

Figure 10. The membership functions of ec^k at the sample step k

For $e^k = -0.25$, Figure 9 shows that the value of membership functions μ_{e-NB} and μ_{e-NM} can be calculated by

$$\mu_{e-NB}(-0.25) = \frac{-0.25+0.1925}{-0.2873+0.1925} = 0.5009,$$

$$\mu_{e-NM}(-0.25) = \frac{-0.25+0.2873}{-0.1925+0.2873} = 0.4991.$$

Obviously, the values for the other four membership functions μ_{e-NS}, μ_{e-PS}, μ_{e-PM}, μ_{e-PB}, are zeroes. For $ec^k = 0.12$, Figure 10 shows that the values of membership functions μ_{ec-PB} and μ_{ec-PM} can be calculated by

$$\mu_{ec-PB}(0.12) = \frac{0.12-0.0995}{0.1485-0.0995} = 0.4184,$$

$$\mu_{ec-PM}(0.12) = \frac{0.12-0.1485}{0.0995-0.1485} = 0.5816.$$

Similarly, the values for the other four membership functions μ_{ec-NB}, μ_{eC-NM}, μ_{ec-NS}, μ_{ec-PS}, are zeroes. At the $(k+1)$th sample step, the duty cycle of PWM can be calculated by

$$\rho^{k+1} = \frac{(0.722)^{0.9}}{0.5009\times0.4184+0.5009\times0.5816+0.4991\times0.4184+0.4991\times0.5816}\bullet$$

$$(0.5009\times0.4184\times0.67+0.5009\times0.5816\times0.67+0.4991\times0.4184\times0.67+0.4991\times0.5816\times0.67)$$

Thus, one gets $\rho^{k+1} = 0.4998$.

To contrast the actual effect of the VUD fuzzy controller and the original controller under the conditions of different input positions, the authors input two position testing instructions (-50mm, -170mm). Under the condition of the input instruction (-50mm), the position data of work piece was plotted in Figure 11 (a), where f50 represents the output response curve of the VUD fuzzy controller and g50 the output response curve of the original controller. Similarly, under the condition of the input instructions (-170mm), the position data of work piece was plotted in Figure 11 (b), where f170 represents the output response curve of the VUD fuzzy controller, g170 the output response curve of the original controller.

To take a closer look at the positioning accuracy of work piece, the authors defined that $fe50 = |-50 - y50|$, $ge50 = |-50 - g50|$, $fe170 = |-170 - f170|$ and $ge170 = |-170 - g170|$. The curves of $fe50$, $ge50$, $fe170$ and $ge170$ at the stable stage $(t > 2.0\text{s})$ are plotted in Figure 12.

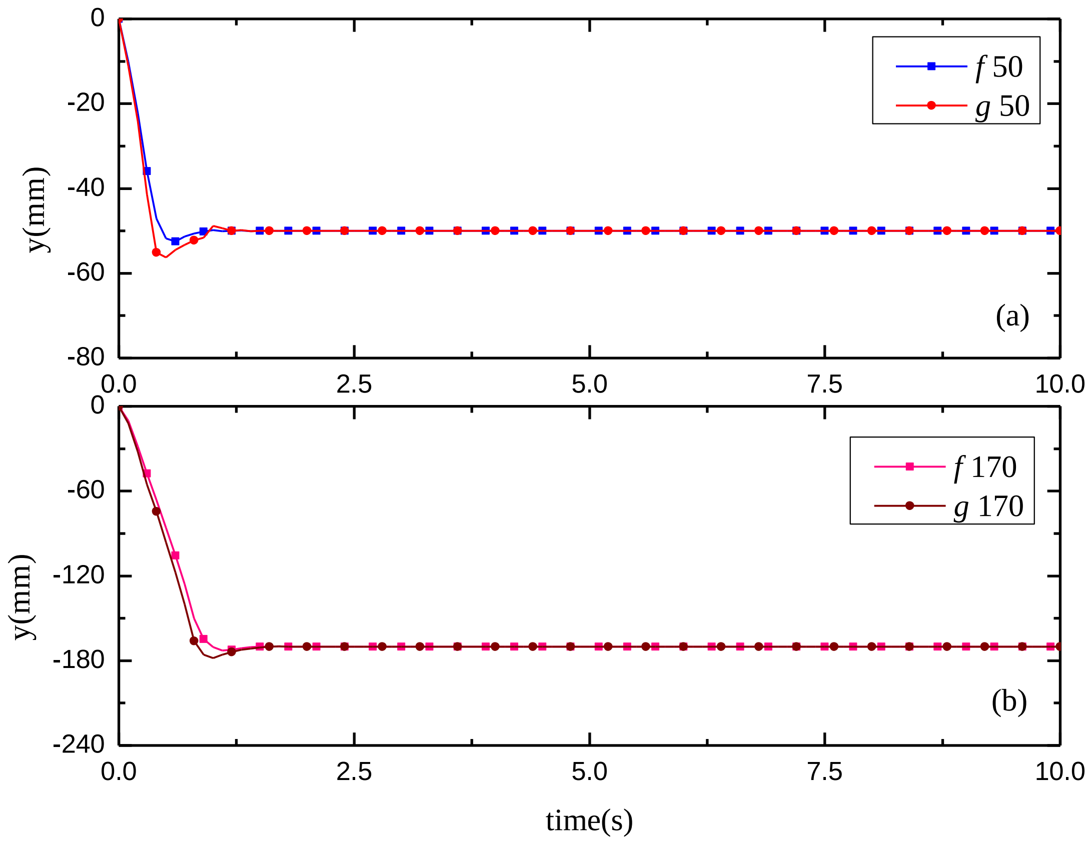

Figure 11. Curve of displacement of work piece

Figure 11 (a) shows that both $f50$ and $g50$ almost reach -50mm at 1.0s. At the same time, Figure 11 (b) shows that both $f170$ and $g170$ almost reach -170mm at 1.25s. It can be found that the original controller has a little quicker response than the VUD fuzzy controller. However, Figure 12 shows that both $fe50$ and $fe170$ locate in the range of $[-2.0\mu\text{m}, 2.0\mu\text{m}]$ and both $ge50$ and $ge170$ in the range of $[-10.0\mu\text{m}, 10.0\mu\text{m}]$. One can easily see that the VUD fuzzy controller gets a significant improvement on the requirements of positioning accuracy, although its response is a little slower than the original controller. Therefore, the VUD fuzzy controller shown as (21) achieves the expected effect.

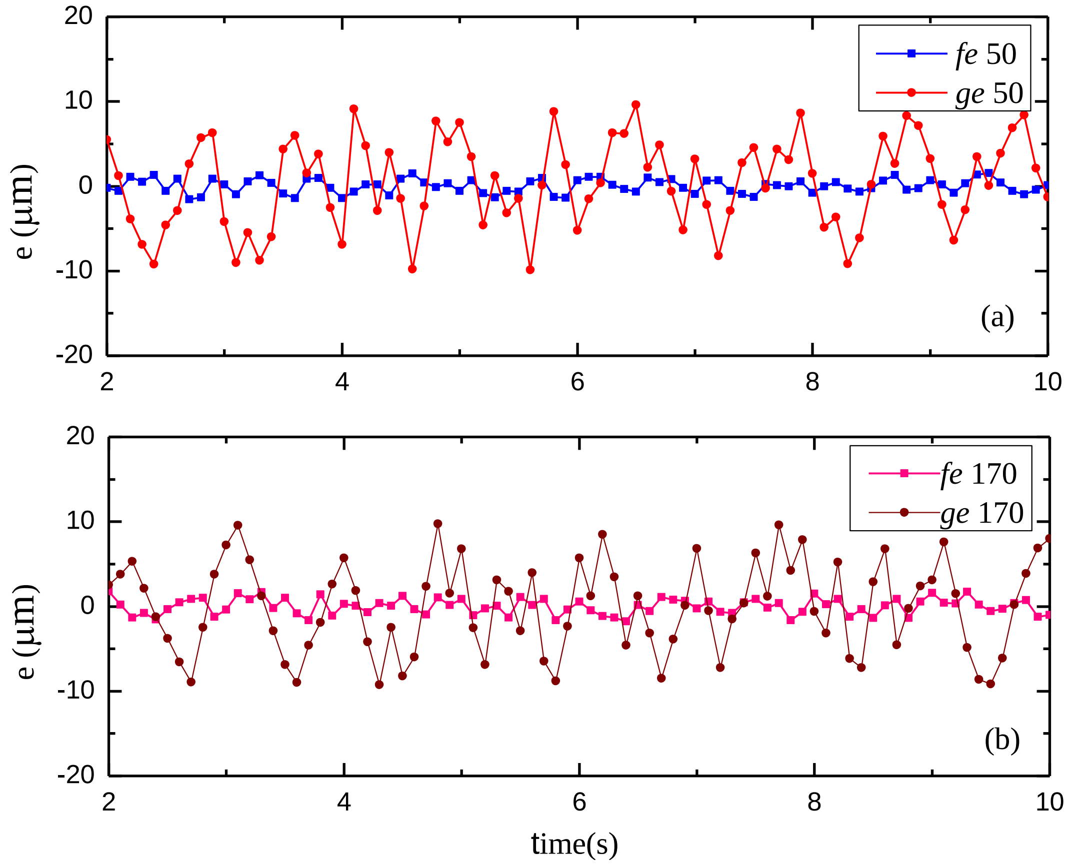

Figure 12. Error curve of displacement of work piece

VI. CONCLUSIONS

To improve the positioning accuracy of work pieces in CNC lathe, a novel VUD fuzzy controller is proposed. For optimum advantage of the VUD fuzzy controller, the photo–electricity encoder in the servo system was replaced by a grating–ruler. A loop system, based on the VUD fuzzy controller, was constructed by five steps. The method of lookup table was used to design the base of fuzzy rules by four typical input instructions. The conflicting rules were properly settled by the maximum strength. The experimental results show that the VUD fuzzy controller achieved good effect in controlling the position of work pieces, and that the error in positioning satisfied the requirements of control accuracy.

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