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LEADERLESS AND LEADER-FOLLOWING FLOCKING MOTION VIA COORDINATED CONTROL

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Abstract- In this paper, novel coordinated control strategies are presented for control and analysis of multi-agents with point mass dynamics to achieve leaderless and leader-following flocking motions. Four control laws are proposed for a group of agents to achieve flocking formations. Two of them are for leaderless flocking and two for leader-following flocking relative to two different centers (mass center and geometric center) of the flock, respectively. A distance-dependent adjacency matrix is used to quantify the way agents influence each other. Stability analysis of the control systems is conducted based on the classical Lyapunov theory to indicate the flocking behaviors (cohesiveness, collision avoidance and velocity matching) of the systems. Finally, simulation examples are given to validate the theoretical results.

Index terms***:*** **Flocking motion, multi-agent, coordinated control.**

I. INTRODUCTION

Flocking motions of a flock of birds, a herd of land animals, a swarm of insects, or a school of fish, can be seen everywhere in the natural world. Scientists from rather diverse disciplines including animal behavior, physics, biophysics, social sciences, and computer science try to understand how they move together coherently[1-6]. The study of these topics has formed an active area of research, giving rise to new control paradigms such as quantized control systems, networked control systems and multi-agent (multi-vehicle, multi-robot) systems.

In 1986, Reynolds [5] developed a computer animation model for flocking motion of groups of interactive agents based on three heuristic rules, i.e., separation, alignment and cohesion. To the best of our knowledge, the first papers which gave a theoretical explanation for Reynolds's model are [7] in fixed topologies and [8] in dynamic topologies, respectively. However, in the dynamic topology case [8], control discontinuities require a stability analysis within the framework of Filippov solutions and non-smooth stability which is difficult to be understood for engineering researchers. In order to overcome these difficulties in theoretical analysis, the smooth Laplacian and the smooth artificial potential field function are introduced by Olfati-Saber in [9] to provide a theoretical and computational framework for design and analysis of scalable flocking algorithms in high-dimensional space in presence or lack of obstacles. Some other works issued on flocking in recent years are [10-13]. Some other researches tightly related this topic are the consensus problem [14-17] and distributed computing [18].

In this paper, the collective behaviors of multi-agent systems with point mass dynamics are investigated in high-dimensional space. The major differences or contributions compared with the existing works, for example [7-9], can be outlined as follows. First of all, the results are based on a more general particle model and the flocking motion is centered at different centers, e.g. the mass center and the geometric center. Secondly, four control laws based on different centers are proposed, such that the desired collective behaviors (flocking centering, collision avoidance and velocity matching) can be achieved. Lastly, a leader agent with time variant dynamics is introduced as a reference model. With respect to different centers of the group of agents respectively, algorithms are gradient-based control laws equipped with a velocity consensus protocol and a regulator based on leader's state information. The gradient-based term is designed to be a vector in the direction of negated gradient of a potential function and will contribute to flocking centering and collisions avoidance in the group. The velocity consensus term and the regulate term aim at aligning the velocity vectors of all the agents and their related center, and to make them move with the same speed and direction as that of the leader. The distance-dependent adjacency matrix is used to quantify the way agents influence each other and leads to a fixed interconnected topology. The theoretical analysis is then convenient using the classical Lyapunov theory.

The rest of the paper is organized as follows. The problem is stated in section II. Control laws for flocking motion based different centers are presented in section III. Our main results are presented in section IV. Simulation results are provided in section V. Finally, concluding remarks are made in section VI.

II. PROBLEM STATEMENT

Consider N agent, moving in R^n with dynamics described by:

$$\begin{aligned} \dot{x}^i &= v^i \\ m_i \dot{v}^i &= u^i - k_i v^i \qquad i = 1, 2, \cdots, N, \end{aligned} \tag{1}$$

where $m_i > 0$ is its mass, $x^i = (x_1^i, x_2^i, \cdots x_n^i)^T \in R^{\mathrm{n}}$ is the position vector of agent i, $v^i = (\dot{x}_1^i, \dot{x}_2^i, \cdots \dot{x}_n^i)^T \in R^{\mathrm{n}}$ is its velocity vector and $u_i = (u_{x_1^i}, u_{x_2^i}, \cdots, u_{x_n^i})^T \in R^{\mathrm{n}}$ is the control (acceleration) input, $k_i > 0$ is the velocity damping gain, and $-k_i v^i$ is the velocity damping term.

It is necessary to consider the case in which the group of mobile agents move with a leader which is driven at a known variant velocity $v^0(t)$. The relative position vector between the ith agent and the leader is denoted as $r_p^i = x^i - x^0$ and the relative velocity vector between the ith agent and the leader is denoted as $r_v^i = v^i - v^0$. The relative position and relative velocity vector between agent i and j are denoted as $r_p^{ij} = r_p^i - r_p^j$ and $r_v^{ij} = r_v^i - r_v^j$, respectively. The motion equations of agent i relative to the leader agent can be expressed as:

$$\begin{aligned} \dot{r}_p^i &= r_v^i \\ m_i \dot{r}_v^i &= u^i - k_i v^i \qquad i = 1, 2, \cdots, N. \end{aligned} \tag{2}$$

The objectives are to achieve flocking behaviors for the whole group and, in leader-following coordination, forcing the velocity vector of the group of agents and their related centers converge to leader's velocity vector. The control input for agent i is a combination of three components:

$$u^i = \alpha^i + \beta^i + \gamma^i + k_i v^i \qquad i = 1, 2, \cdots, N. \tag{3}$$

The first component α^i is derived from the field produced by an artificial potential field function, which depends on the relative distances between agent i and its flockmates. This term is responsible for collision avoidance and cohesion in the group. The second component β^i aligns the velocity vector of agent i. The third component γ^i is a regulated term based on leader's state information.

The motion of a group of mobile agents is said to flocking motion, when all agents attain the same velocity vector, distances between the agents are stabilized, and no collisions between them occur. The problem is to design the control input (3) so that the group of mobile agents achieves flocking motion.

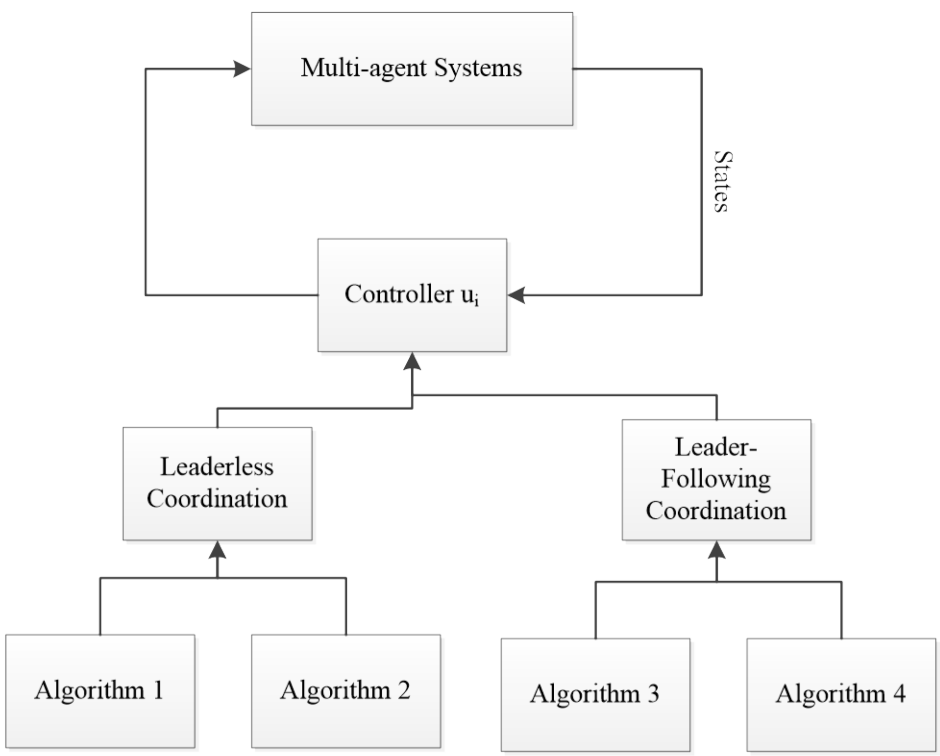

Figure 1. Problem decomposition

III. COORDINATION STRATEGIES

In this section, we will refine the control input (3) into specific expressions for the components α^i, β^i and γ^i . Four algorithms are proposed for leaderless and leader-following coordination, respectively. Figure 1 shows the problem decomposition.

III.1. Leaderless Coordination

In this subsection, for leaderless coordination, two control laws are proposed for the mass center and the geometric center, respectively.

Algorithm 1:

The mass center of all agents is defined as:

$$x_{mc} = \sum_{i=1}^{N} m_i x^i \Big/ \sum_{i=1}^{N} m_i . \tag{4}$$

The control law based on mass center is then designed as:

$$u^i = \underbrace{-\sum_{j=1}^{N} a_{ij} \nabla_{x^i} U_{ij}}_{\alpha^i} \underbrace{-\sum_{j=1}^{N} a_{ij} (v^i - v^j)}_{\beta^i} + k_i v^i, \quad i = 1, 2, \cdots, N. \tag{5}$$

Algorithm 2:

The geometric center of all agents is defined as

$$x_{gc} = ave(x) = \frac{1}{N} \sum_{i=1}^{N} x^i . \tag{6}$$

The control law based on geometric center is then defined as

$$u^i = \underbrace{-\sum_{j=1}^{N} a_{ij} m_i \nabla_{x^i} U_{ij}}_{\alpha^i} \underbrace{-\sum_{j=1}^{N} a_{ij} m_i (v^i - v^j)}_{\beta^i} + k_i v^i, \quad i = 1, 2, \cdots, N, \tag{7}$$

where the weights a_{ij} quantify the way agents influence each other. It is reasonable to assume that the influence between agents is a function of the relative distance between agents. Similar to [19], a form is given to this assumption via a nonincreasing function $\eta : R^+ \to R^+$ such that the adjacency matrix A has entries:

$$a_{ij} = \eta(\|x^i - x^j\|^2) . \tag{8}$$

An example of nonincreasing function $\eta(\cdot)$ is:

$$\eta(y) = \frac{K}{(\sigma^2 + y)^s}, \tag{9}$$

where $K, \sigma > 0$ and $s \geq 0$ are fixed constants.

Function U_{ij} depends on the relative distance between the agents and defined as follows.

Definition (Potential field function) [7] Potential field function U_{ij} is a differentiable, nonnegative, radially unbounded function of the distance $\|x^i - x^j\|$ between agents i and j , such that

(1) $U_{ij}(\|x^i - x^j\|) \to \infty$ as $\|x^i - x^j\| \to 0$;

(2) U_{ij} attains its unique minimum when agents i and j are located at a desired distance;

Cohesion and separation is achieved using artificial potential fields. One possible choice could be:

$$U_{ij}(\|x^i - x^j\|) = c\left[\frac{d^2}{\|x^i - x^j\|^2} + \ln\|x^i - x^j\|^2\right], \tag{10}$$

where $c, d > 0$ are constant.

Let

$$\psi(z) = c[\frac{d^2}{z^2} + \ln(z^2)],$$

then

$$\varphi(z) = \frac{d\psi(z)}{dz} = \frac{2c(z^2 - d^2)}{z^3}.$$

Therefore, the potential field function U_{ij} reaches its unique minimum at $\|x^i - x^j\| = d$. Based on this potential function, control law in (5) and (7) can be rewritten as:

$$u^i = \underbrace{-\sum_{j=1}^{N} a_{ij}\phi(\|x^i - x^j\|)\vec{n}_{ij}}_{\alpha^i} \underbrace{-\sum_{j=1}^{N} a_{ij}(v^i - v^j)}_{\beta^i} + k_i v^i, \quad i = 1, 2, \cdots, N, \tag{11}$$

and

$$u^i = \underbrace{-\sum_{j=1}^{N} a_{ij} m_i \phi(\|x^i - x^j\|)\vec{n}_{ij}}_{\alpha^i} \underbrace{-\sum_{j=1}^{N} a_{ij} m_i (v^i - v^j)}_{\beta^i} + k_i v^i, \quad i = 1, 2, \cdots, N, \tag{12}$$

where $\vec{n}_{ij} = x^i - x^j \big/ \|x^i - x^j\|$ is an unit vector along the line connecting x^j to x^i.

Having defined U_{ij}, the total potential of agent i is expressed as:

$$U_i = \sum_{j=1}^{N} a_{ij} U_{ij}(\left\| x^i - x^j \right\|). \tag{13}$$

III.2. Leader-Following Coordination

In this subsection, for leader-following coordination, two types of control laws of agent i are proposed with respect to the mass center and the geometric center, respectively.

Algorithm 3:

For the mass center of agents defined in (4), and the equations of leader-following motion of agent i derived in (2), the control law is designed as:

$$u^i = \underbrace{-\sum_{j=1}^{N} a_{ij} \nabla_{r_p^i} U_{ij}(\left\| r_p^{ij} \right\|)}_{\alpha^i} \underbrace{-\sum_{j=1}^{N} a_{ij}(r_v^i - r_v^j)}_{\beta^i} \underbrace{-m_i r_v^i + m_i \dot{v}_0}_{\gamma^i} + k_i v^i, \quad i = 1, 2, \cdots, N. \tag{14}$$

Algorithm 4:

For geometric center of agents defined in (6), and the equations of leader-following motion of agent i derived in (2), the control law is designed as:

$$u^i = \underbrace{-\sum_{j=1}^{N} a_{ij} m_i \nabla_{r_p^i} U_{ij}(\left\| r_p^{ij} \right\|)}_{\alpha^i} \underbrace{-\sum_{j=1}^{N} m_i a_{ij}(r_v^i - r_v^j)}_{\beta^i} \underbrace{-m_i r_v^i + m_i \dot{v}_0}_{\gamma^i} + k_i v^i, \quad i = 1, 2, \cdots, N. \tag{15}$$

By the definition of U_{ij}, we have

$$U_{ij}(\left\| x^i - x^j \right\|) = U_{ij}(\left\| r_p^{ij} \right\|) = U_{ij}(\left\| r_p^i - r_p^j \right\|)$$

and

$$\nabla_{x^i} U_{ij}(\left\| x^i - x^j \right\|) = \nabla_{r_p^i} U_{ij}(\left\| r_p^{ij} \right\|).$$

Based on the potential function (10), the control laws in (14) and (15) can be rewritten as:

$$u^i = \underbrace{-\sum_{j=1}^{N} a_{ij} \phi(\left\| r_p^{ij} \right\|) \vec{n}_{ij}}_{\alpha^i} \underbrace{-\sum_{j=1}^{N} a_{ij}(r_v^i - r_v^j)}_{\beta^i} \underbrace{-m_i r_v^i + m_i \dot{v}^0}_{\gamma^i} + k_i v^i, \quad i = 1, 2, \cdots, N. \tag{16}$$

and

$$u^i = \underbrace{-\sum_{j=1}^{N} a_{ij} m_i \phi(\|r_p^{ij}\|)\vec{n}_{ij}}_{\alpha^i} \underbrace{-\sum_{j=1}^{N} a_{ij} m_i (r_v^i - r_v^j)}_{\beta^i} \underbrace{- m_i r_v^i + m_i \dot{v}^0}_{\gamma^i} + k_i v^i, \quad i = 1, 2, \cdots, N. \tag{17}$$

where $\vec{n}_{ij} = r_p^{ij} \big/ \|r_p^{ij}\|$ is an unit vector along the line connecting x^j to x^i.

Having defined U_{ij}, the total potential of agent i can be written as

$$V_i = \sum_{j=1}^{N} a_{ij} U_{ij}(\|x^i - x^j\|) = \sum_{j=1}^{N} a_{ij} U_{ij}(\|r_p^i - r_p^j\|) = \sum_{j=1}^{N} a_{ij} U_{ij}(\|r_p^{ij}\|). \tag{18}$$

IV. MAIN RESULTS

In this section, we give the stability results of multiple mobile agents with point mass dynamics described by (1) and (2).

IV.1. Leaderless Flocking

IV.1.1. Flocking motion based on mass center

By definition of x_{mc} in (4), we have

$$\begin{cases} \dot{x}_{mc} = v_{mc} \\ \dot{v}_{mc} = \sum_{i=1}^{N} (u_{mc}^i - k_i v_i) \Big/ \sum_{i=1}^{N} m_i . \end{cases} \tag{19}$$

By symmetry of the adjacency matrix A and the potential field function U_{ij} respect to $x^i - x^j$, we have $\dot{v}_{mc} = 0$, i.e., the velocity of mass center of the flock is invariant as time evolves and equal to $v_{mc}(0) = \sum_{i=1}^{N} m_i v^i(0) \Big/ \sum_{i=1}^{N} m_i$.

Consider the following positive semi-definite function:

$$Q = \frac{1}{2} \sum_{i=1}^{N} (U_i + m_i v^{i\mathrm{T}} v^i). \tag{20}$$

Applying LaSalle's invariant principle, it shows that the closed loop system of agents (1) using algorithm 1 achieves flocking motion asymptotically.

Theorem 1 Consider a system of N mobile agents with dynamics(1), each steered by control law (5). Then all agent velocity vectors become asymptotically the same as the velocity vector of mass center, the velocity vector of mass center is invariant as time evolves and equal to $v_{mc}(0)$, the relative distance between agents maintain constant, collisions between agents are avoided, and the system approaches a configuration that minimizes all agent potentials.

The proof process of theorem 1 is similar to that of theorem IV.4 in [7] and omitted here.

IV.1.2 Flocking motion based on geometric center

By definition of x_{gc} in (6),

$$\begin{cases} \dot{x}_{gc} = v_{gc} \\ \dot{v}_{gc} = \sum_{i=1}^{N} \frac{1}{m_i}(u_{gc}^i - k_i v_i) \Big/ N. \end{cases} \quad (21)$$

Similarly, it is obtained $\dot{v}_{gc} = 0$, i.e., the velocity vector of geometric center of the flock is invariant as time evolves and equal to $v_{gc}(0) = \frac{1}{N}\sum_{i=1}^{N} v^i(0)$.

Consider following positive semi-definite function:

$$W = \frac{1}{2}\sum_{i=1}^{N}(U_i + v^{i\mathrm{T}} v^i) \quad (22)$$

Applying LaSalle's invariant principle, it shows that the closed loop system of agents (1) using algorithm 2 achieves flocking motion asymptotically.

Theorem 2 Consider a system of N mobile agents with dynamics(1), each steered by control law (7). Then all agent velocity vectors become asymptotically the same as the velocity vector of geometric center, the velocity vector of geometric center is invariant as time evolves and equal to $v_{gc}(0)$, the relative distance between agents maintain constant, collisions between agents are avoided, and the system approaches a configuration that minimizes all agent potentials.

The proof of theorem 2 is similar to that of theorem IV.4 in [7] and omitted here.

IV.2 Leader-Following Flocking

IV.2.1 Flocking motion based on mass center

By definition of x_{mc} in (4),

$$\begin{cases} \dot{x}_{mc} = v_{mc} \\ \dot{v}_{mc} = \sum_{i=1}^{N} (u_{mc}^{i} - k_i v^i) \Big/ \sum_{i=1}^{N} m_i \, . \end{cases} \tag{23}$$

Similarly, it is obtained $\dot{v}_{mc}(t) = -v_{mc}(t) + v^0(t) + \dot{v}^0(t)$ and then $v_{mc}(t) = v^0(t) + (v_{mc}(0) - v^0(0))e^{-t}$. Therefore, the velocity of mass center of the flock exponentially converges to the leader's velocity $v^0(t)$ as time evolves.

Consider following positive semi-definite function:

$$Q = \frac{1}{2}\sum_{i=1}^{N}(V_i(\left\| r_p^i - r_p^j \right\|) + m_i r_v^{i\mathrm{T}} r_v^i) \tag{24}$$

Applying LaSalle's invariant principle, it shows that the closed loop system of agents (2) using algorithm 3 achieves flocking motion asymptotically.

Theorem 3 Consider a system of N mobile agents with dynamics (2), each steered by control law (14). Then all agent velocity vectors become asymptotically the same as leader's velocity vector, the velocity vector of mass center exponentially converges to leader's velocity vector, collisions between agents are avoided, and the system approaches a configuration that minimizes all agent potentials defined in (24).

The proof of theorem 3 is similar to that of theorem IV.4 in [7] and omitted here.

IV.2.2 Flocking motion based on geometric center

By definition of x_{gc} in (6), it has

$$\begin{cases} \dot{x}_{gc} = v_{gc} \\ \dot{v}_{gc} = \sum_{i=1}^{N} \frac{1}{m_i} (u_{gc}^{i} - k_i v^i) \Big/ N \, . \end{cases} \tag{25}$$

We have $\dot{v}_{gc}(t) = -v_{gc}(t) + v^0(t) + \dot{v}^0(t)$, and then $v_{gc}(t) = v^0(t) + (v_{gc}(0) - v^0(0))e^{-t}$. It follows that the velocity vector of geometric center of the flock exponentially converges to the leader's velocity vector $v^0(t)$ as time evolves.

Consider following positive semi-definite function:

$$W = \frac{1}{2}\sum_{i=1}^{N}(V_i(\left\| r_p^i - r_p^j \right\|) + r_v^{i\mathrm{T}} r_v^i) \tag{26}$$

Applying LaSalle's invariant principle, it shows that the closed loop system of agents (2) using algorithm 4 achieves flocking motion asymptotically.

Theorem 4 Consider a system of N mobile agents with dynamics (2), each steered by control law (15). Then all agent velocity vectors become asymptotically the same as leader's velocity, the velocity vector of geometric center exponentially converges to leader's velocity, collisions between agents are avoided, and the system approaches a configuration that minimizes all agent potentials defined in (26).

The proof of theorem 4 is similar to that of theorem IV.4 in [7] and omitted here.

V. SIMULATION EXAMPLES

V.1. Leaderless Flocking Example

In this subsection, the situation of leaderless flocking motion based on mass center, i.e., Algorithm 1, was simulated in 2-dimentional space. The following parameters were fixed throughout the simulations: $c=1$, $d=5$ for $\phi(z)$, and $K=4$, $\sigma=2$, $s=0.4$ for $\eta(y)$. The simulation was calculated in 20 seconds by using Matlab Simulink. In addition, the position of each agent was marked with a pentagram sign, the position of the mass center was marked with a red one.

Figures 2 to 6 show the simulation results within 2-D flocking using Algorithm 1 for 50 agents. Figures 2 to 4 show snapshots of 2-D flocking at time 0, 9.8471, and 20. The initial positions were chosen randomly from a normal distribution with mean 0 and variance 1600. The initial velocity coordinates were uniformly chosen in a random domain of $[1,5]\times[1,5]$. The mass of each agent was also uniformly chosen in a random domain of $[1,2]$. A steady configuration was formed as shown in Figure 4 and maintained thereafter. Figure 5 shows the velocities of all agents converging to the velocity of mass center (red line) is achieved along x-axis and y-axis respectively. Figure 6 shows the trajectories of all agents and mass center marked with red circles in simulation time and the cohesive behaviors.

The experimental results show that all agent velocity vectors become asymptotically the same as the velocity vector of mass center, the velocity vector of mass center is invariant as time evolves and equal to $v_{mc}(0)$, the relative distance between agents maintain constant, collisions between agents are avoided, and the system approaches a configuration that minimizes all agent potentials. The simulation demonstration with Algorithm 2 was similar to that conducted by Algorithm 1, and therefore is not necessarily repeated here.

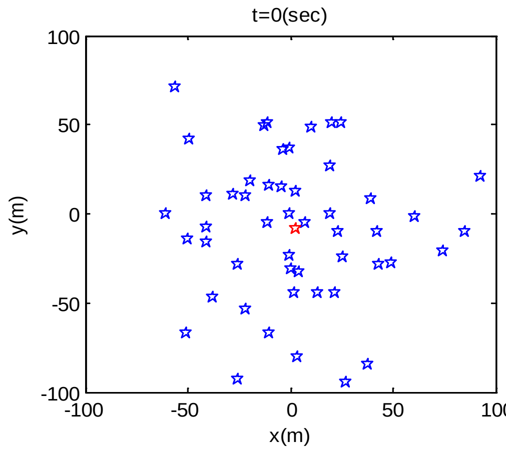

Figure 2. Initial positions of 50 agents

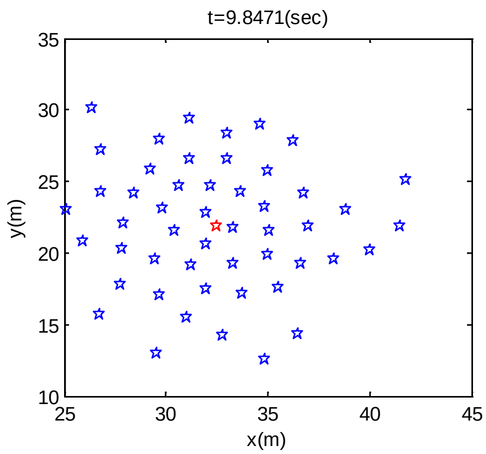

Figure 3. Configuration of 50 agents at t=9.8471(sec)

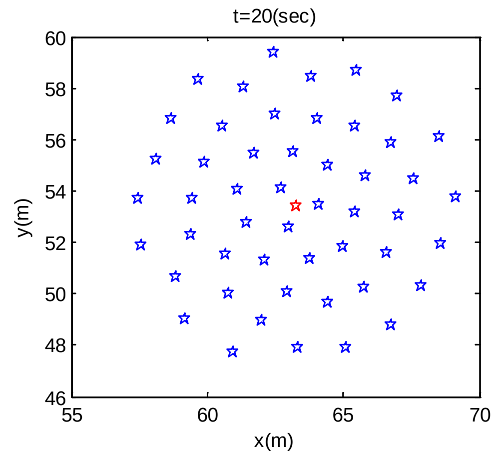

Figure 4. Final configuration of 50 agents at t=20 (sec)

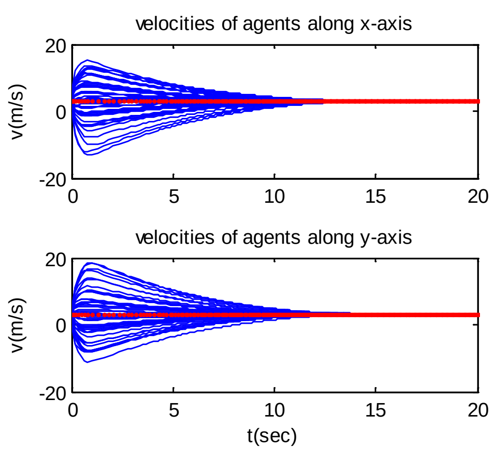

Figure 5. Velocities of agents along x-axis and y-axis respectively

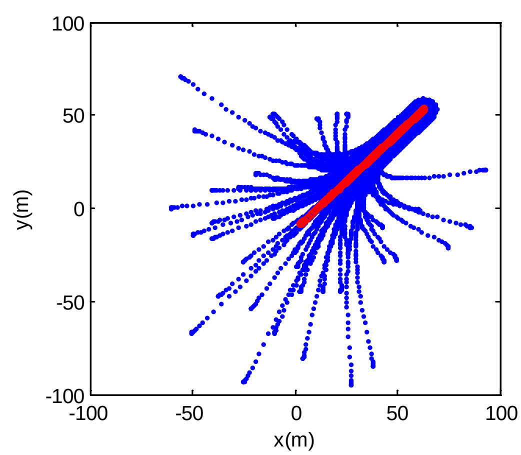

Figure 6. Trajectories of all agents in 20 (sec) time

V.2. Leader-Following Example

In this section, the situation of leader-following flocking motion based on mass center, i.e., algorithm 3, was simulated in 2-dimentional space. The following parameters were fixed throughout the simulations: $c=1$, $d=5$ for $\phi(z)$, and $K=4$, $\sigma=2$, $s=0.4$ for $\eta(y)$ and let the motion trajectory of the leader be $\begin{cases} x_1^0(t)=2t \\ x_2^0(t)=0.1t^2 \end{cases}$. The simulation was calculated in 50 seconds time by using Matlab Simulink. In addition, the position of each agent was marked with a pentagram sign, the position of the mass center was marked with a red pentagram sign.

Figure 7 to 12 shows the simulation results within 2-D flocking using algorithm 3 for 50 agents. Figure 7 to 10 show snapshots of 2-D flocking at time 0, 10.5446, 27.662, and 50. The initial positions were chosen randomly from a normal distribution with mean 0 and variance 1600. The initial velocity coordinates were uniformly chosen in a random domain of $[1,5]\times[1,5]$. The mass of each agent was also uniformly chosen in a random domain of $[1,2]$. A steady configuration was formed as shown in Figure 10 and maintained thereafter. Figure 11 shows the velocities of all agents converging to the velocity of mass center (red line) is achieved along x-axis and y-axis respectively. Figure 12 shows the trajectories of all agents, mass center marked with red circles and leader agent marked with yellow stars in simulation time and the cohesive behaviors.

The experimental results show that all agent velocity vectors become asymptotically the same as leader's velocity vector, the velocity vector of mass center exponentially converges to leader's velocity vector, collisions between agents are avoided, and the system approaches a configuration that minimizes all agent potentials.

The simulation demonstration with algorithm 4 was similar to that conducted by algorithm 3, and therefore is not necessarily repeated here.

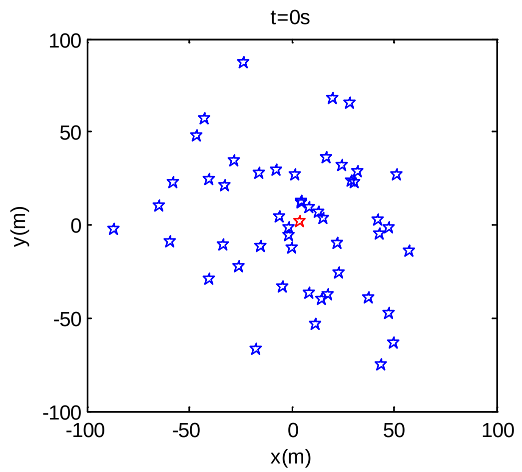

Figure 7. Initial positions of 50 agents

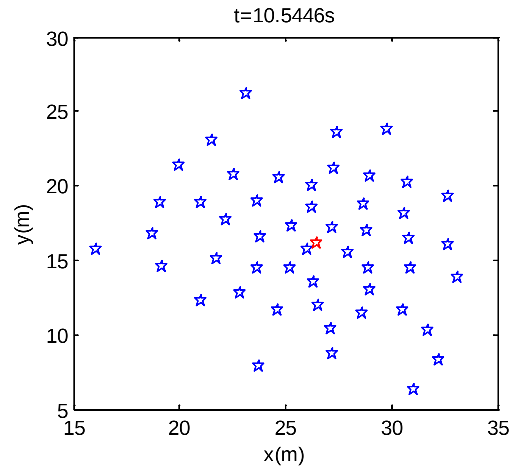

Figure 8. Configuration of 50 agents at t=10.5446 (sec)

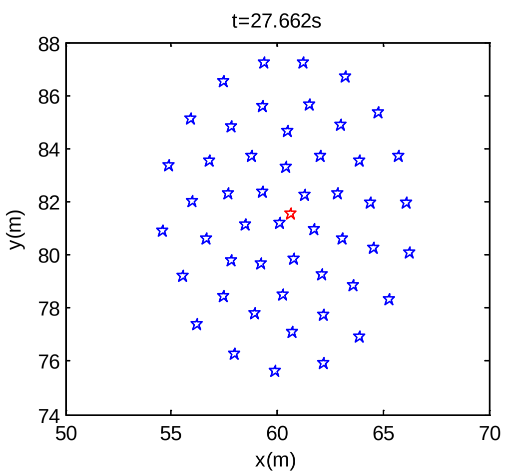

Figure 9. Configuration of 50 agents at t=27.662 (sec)

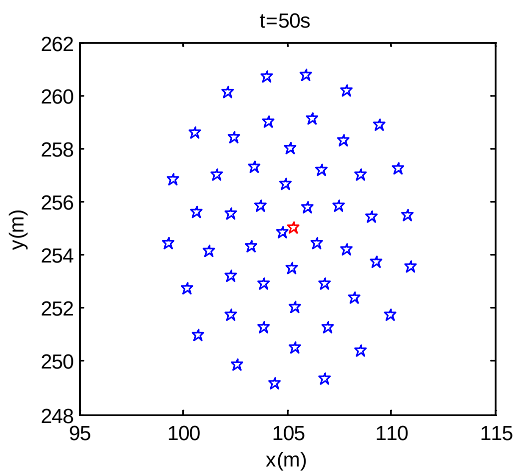

Figure 10. Final configuration at t=50(sec)

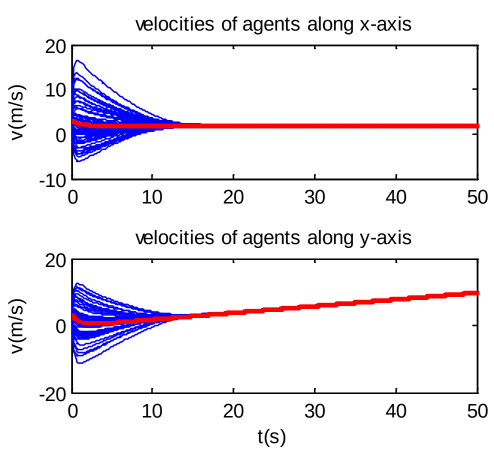

Figure 11. Velocities of agents along x-axis and y-axis respectively

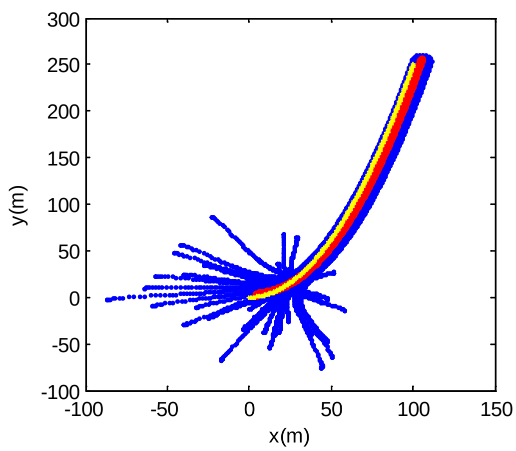

Figure 12. Trajectories of all agents in 50 (sec) time

VI. CONCLUSIONS

This paper establishes a theoretical framework for the design and analysis of flocking algorithms for multiple agent dynamic systems in a high-dimensional space. Control laws have been proposed for dynamic multi-agents to achieve flocking motion relative to different centers. A distance-dependent adjacency matrix is used to model the interconnected relationship between agents. Stability analysis is done using classical Lyapunov theory. Simulation example is tested to validate the theoretical results.

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