

# A New Non-Singular Terminal Sliding Mode Control and Its Application to Chaos Suppression in Interconnected Power System

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**Abstract**—Interconnected power system is a typical nonlinear dynamical system, it will cause great harm to the interconnected power system when the chaos occurred. This paper analyses the nonlinear dynamical behavior of the interconnected power system with a uncertain electromagnetic disturbance amplitude, and the influence of the electromagnetic disturbance amplitude on the stability of the system is obtained. Thus, this paper proposes a novel non-singular terminal sliding mode control to restrain the chaos, then the system will reach a stable state in a fixed time. According to the theoretical analysis, by introducing the saturation function, the control method can solve the singularity problem in the sliding mode control, and the interconnected power system will be stable in a short time when it's in chaos. The simulation prove the correctness of the method.

**Keywords**-Interconnected Power System; Saturation Function; Non-singularity; Sliding Mode Control; Fixed Time

## I. INTRODUCTION

Power system is a kind of nonlinear dynamical system with multi-degree of freedom, strong coupling and multi-variable, which has rich dynamical behaviors. With the development of power grid system, grid interconnection has became a inevitable trend, and it can improve the quality of power. At the same time, it also provides convenience for the dispatching optimization of power system[1-3]. However,

chaos often appears in the interconnected power system, it will bring great challenge to the stability of power system. For example, as the result of chaotic oscillations[4-6], several large area power outages appear in the United States, China and Canada in 1955. And it is difficult to suppress this phenomenon by using linear controller[7]. Therefore, it is necessary to study the mechanism of chaos in interconnected power systems and it is meaningful to design the nonlinear controller to suppress the chaos.

Due to the high nonlinear characteristics of the interconnected power system, its stability is very sensitive to outside disturbance. If the perturbation is too large, its operating point will change obviously. There are also some tools to analyze its stability which include geometric method, energy function, bifurcation theory and numerical simulation[8]. Recently, there are a lot of scholars to study the stability of power grid system. For example, Nayfeh [9] uses the multi-scale perturbation method to study the stability of single machine power system and the bifurcation analysis of a single machine infinite power system is investigated by Duan[10]. Because coupling power angle exists in the interconnected power system, the inherent dynamical behavior of interconnected power system will be more abundant. Through the detailed numerical simulation, the influence of the conventional non-linearity index on the dynamic characteristics of the interconnected power system is expounded[11].

In recent years, with the development of control method, the system is controlled from the single machine infinite system to multiple machine system, and the interconnected power system. In this paper, by proposing a fix-time non-singular terminal sliding mode control to realize stability when chaos occurred in interconnection power system. Some systems can be stabilized in a fixed time by using finite time control, this control method can be used in a lot of fields (for instance[12-13]). However, it is difficult to ensure the boundary convergence time when it is independent of the initial state. Therefore, in some practical system, it is not workable to use this control method when the initial condition is uncertain. Fortunately, this question was solved by Polyakov with the fixed-time stability theory[14]. Zuo[15] proposed a non-singular fixed-time terminal sliding mode controller for a class of second order nonlinear systems that can solve the singularity problem of terminal sliding mode controller in most instances. In this paper, by using the fast terminal sliding mode control can make the system convergence to steady state in finite time, the saturation function[16] and fast fixed time stability theory that can solve the singular problem through theoretical proof in this control method. This control method not only solve the singularity problem of the sliding mode controller, but the convergence speed is faster.

Motivated by the above analysis, this paper investigated the dynamical characteristics and control of interconnected power system. Section 2 introduces the dynamical characteristics of the interconnected power system, by introducing the maximum Lyapunov index, power spectrum, phase diagram and timing diagram, the paper describe the dynamic of the interconnected power system. when the amplitude of electromagnetic disturbance is  $v=1.3$ , the system is in chaos. a non-singular sliding mode variable structure control method has been introduced in Section 3 and the effectiveness of the control method can be obtained by theoretical analysis, the advantages of this method are verified, and the convergence time is calculated in Section 4. And Section 5 gives the conclusion of this paper.

## II. THE ANALYSIS OF THE MODEL AND DYNAMIC CHARACTERISTICS OF THE INTERCONNECTED POWER SYSTEM

There are two kinds of oscillation modes in interconnection power system, one is a single generator acts on other generators in the system with the frequency is between 0.5 and 2.0 Hz. The other oscillation mode is mainly expressed as a generator group in a region interacts with a generator group in another area, the frequency is between 0.1-1.0 Hz. This paper studies the interconnected power system model with two generators. Considering the influence of the amplitude of the electromagnetic disturbance power, the model is as follows:

$$\begin{aligned} \frac{d\delta(t)}{dt} &= \omega(t), \\ \frac{d\omega(t)}{dt} &= -\frac{1}{H}[P_s \sin(\delta(t)) + D\omega(t) - P_m + P_k \cos(\alpha t) \sin(\delta(t)) - P_e \cos(\beta t)] \end{aligned} \quad (1)$$

Where,  $\delta(t)$  is the phase angle between the excitation potential and the terminal voltage between the two generators, and  $\omega(t)$  is the angular velocity of the two generators.  $D$  is the equivalent damping coefficient.

$P_s, P_m, P_e, P_k$  represents the amplitude of the electromagnetic power, the mechanical power, the load disturbance power, and the electromagnetic disturbance power, respectively.  $\alpha, \beta$  is the electromagnetic power disturbance frequency and the load disturbance frequency.  $H$  is the equivalent moment of inertia.

In order to analyses the dynamics of the interconnected power system, the simplified model is obtained:

$$\begin{aligned} \frac{dx_1(\tau)}{d\tau} &= x_2(\tau), \\ \frac{dx_2(\tau)}{d\tau} &= -\sin(x_1(\tau)) - \lambda x_2(\tau) + \rho - v \cos(\chi \tau) \sin(x_1(\tau)) + \mu \cos(\zeta \tau) \end{aligned} \quad (2)$$

Where

$$\begin{aligned} x_1(\tau) &= \delta(t), x_2(\tau) = \omega(t) \sqrt{H/P_s}, \lambda = D / \sqrt{HP_s}, \rho = P_m / P_s, v = P_k / P_s, \\ \mu &= P_e / P_s, \chi = \alpha \sqrt{H/P_s}, \zeta = \beta \sqrt{H/P_s}, \tau = t \sqrt{P_s/H} \end{aligned}$$

Here, the parameters can be selected as  $\lambda=0.4, \mu=0.02, \rho=0.2, \chi=\eta=\zeta=0.8$ .

### A. Lyapunov exponent

The Lyapunov exponent is an important parameter of the system that can be used to measure and determine whether

the dynamic system is in the chaos. When the maximum Lyapunov exponent of the dynamic system is greater than 0, it can be concluded that the system is in chaotic oscillation state.

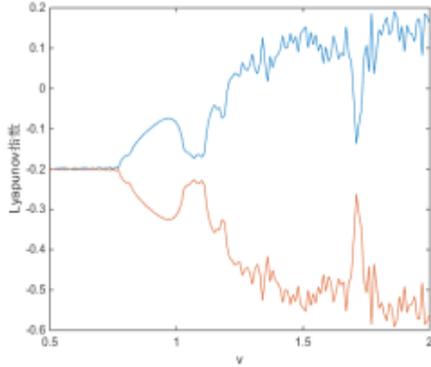


Figure 1. The Lyapunov exponent

### B. Power spectrum

To study the chaotic behavior of a system, it is effective to use power spectrum analysis methods. Actually, power spectrum analysis is through the time and space translate to the frequency space for the signal frequency structure. When the chaos occurs in the system, the power spectrum of the system behaves as a continuous irregular distribution, for example in fig.2(c).

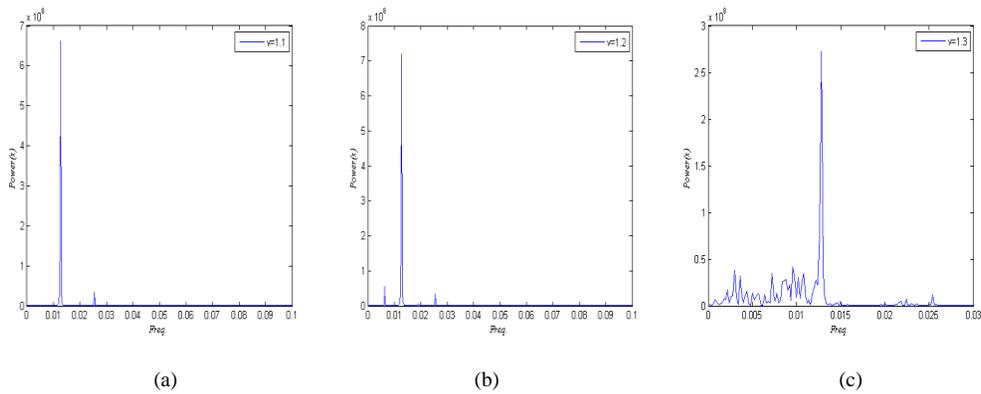


Figure 2. Power spectrum

Similarly, the phase space map also is a tool to determine whether the system is in chaos.

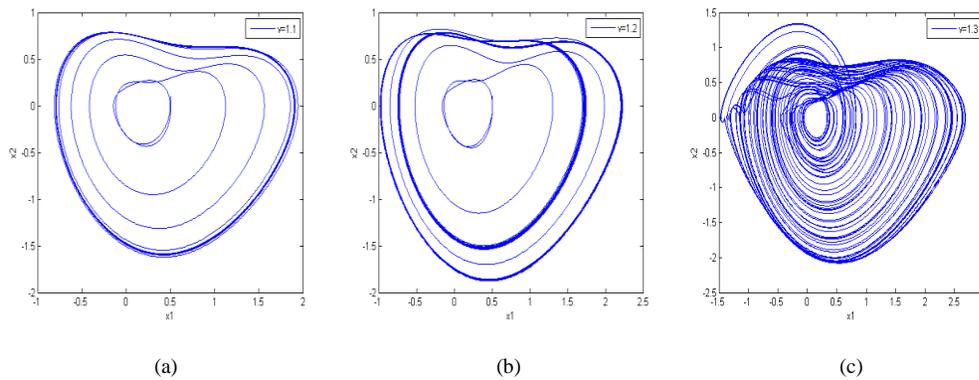


Figure 3. Phase plane plots

According to the maximum Lyapunov exponent, phase diagram, and power spectrum of interconnected power system, we can know that the interconnected power system is in an irregular non-periodic chaotic state when the parameter  $\nu = 1.3$ . This chaotic state will cause great harm to the stability of the system, it will lead to a large area of power outages. Therefore, it is necessary to study the method to restrain the chaos in interconnected power system.

### III. THE SLIDING MODE CONTROLLER DESIGN

In this section, the paper proposes a control scheme that can restrain chaos in power system, this paper needs to add control law in the second item in control equation that can make the system output  $x_1$  convergences to the control target, namely,

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = f(x) + g(x, t) + d(t) \cdot u, \end{cases} \quad (3)$$

Where :

$$\begin{aligned} f(x) &= -\sin x_1 - \lambda x_2 + \rho & g(x, t) &= -\nu \cos(\eta t) \sin x_1 \\ d(t) &= \mu \cos(\gamma t) \end{aligned}$$

By proposing a timing non-singular fast terminal control method, the system can be smoothly stabilized on the sliding surface  $s$ . The sliding surface  $s$  is designed as:

$$s = \dot{x}_1 + \alpha_1 x_1^{\frac{1}{2} + \frac{m_1}{2n_1} + (\frac{m_1}{2n_1} - \frac{1}{2}) \text{sign}(|x_1| - 1)} + \beta_1 x_1^{q_1} \quad (4)$$

the paper can get the reach law :

$$\dot{s} = -\alpha_2 s^{\frac{1}{2} + \frac{m_2}{2n_2} + (\frac{m_2}{2n_2} - \frac{1}{2}) \text{sign}(|s| - 1)} - \beta_2 s^{\frac{p_2}{q_2}} \quad (5)$$

Here, after reading the relevant literature [18] about how to overcome the singularity in terminal sliding mode control.

This paper quotes saturation function that can solve the question.

$$\begin{aligned} u &= -\frac{1}{dt} [f(x) + \alpha_1 (\frac{1}{2} + \frac{m_1}{2n_1} + \\ & (\frac{m_1}{2n_1} - \frac{1}{2}) \text{sign}(|x_1| - 1)) x_1^{\frac{1}{2} + \frac{m_1}{2n_1} + (\frac{m_1}{2n_1} - \frac{1}{2}) \text{sign}(|x_1| - 1)} x_2] \\ & + \text{sat}(\beta_1 \frac{p_1}{q_1} x_1^{q_1} x_2, h) \\ & + \alpha_2 s^{\frac{1}{2} + \frac{m_2}{2n_2} + (\frac{m_2}{2n_2} - \frac{1}{2}) \text{sign}(|s| - 1)} + \beta_2 s^{\frac{p_2}{q_2}} \end{aligned} \quad (6)$$

In control input, the paper quotes saturation function to limit the amplitude of singularity term  $x_1^{q_1} x_2$ , where the saturation function is

$$\text{sat}(x, y) = \begin{cases} x, & \text{if } |x| < y, \\ y \cdot \text{sign}(x), & \text{if } |x| \geq y, \end{cases} \quad (7)$$

#### Theorem 1

In control law(6), if there is a positive number  $\alpha_1, \alpha_2, \beta_1, \beta_2$ , and also  $m_1, n_1, m_2, n_2, p_1, q_1, p_2, q_2$  is odd positive integers satisfying  $m_1 > n_1, m_2 > n_2, p_1 < q_1, p_2 < q_2$ , and  $(m_1 + n_1)/2, (m_2 + n_2)/2, (p_1 + q_1)/2, (p_2 + q_2)$  is odd positive integers. The system 11 will be stable in a fixed time.

*proof*

$$V_1 = \frac{1}{2} s^2 \quad (8)$$

$$\begin{aligned} \dot{V}_1 &= s \cdot \dot{s} = -s(\alpha_2 s^{\frac{1}{2} + \frac{m_2}{2n_2} + (\frac{m_2}{2n_2} - \frac{1}{2}) \text{sign}(|s| - 1)} + \beta_2 s^{\frac{p_2}{q_2}}) \\ &= -\alpha_2 s^{\frac{1}{2} + \frac{m_2}{2n_2} + (\frac{m_2}{2n_2} - \frac{1}{2}) \text{sign}(|s| - 1) + 1} - \beta_2 s^{\frac{p_2}{q_2} + 1} \\ &= -\alpha_2 (2V_1)^{\frac{\frac{1}{2} + \frac{m_2}{2n_2} + (\frac{m_2}{2n_2} - \frac{1}{2}) \text{sign}(|s| - 1) + 1}{2}} - \beta_2 (2V_1)^{\frac{\frac{p_2}{q_2} + 1}{2}} \end{aligned} \quad (9)$$

When  $|s| \geq 1$

$$\dot{V}_1 = -\alpha_2 (2V_1)^{\frac{\frac{m_2+1}{n_2}}{2}} - \beta_2 (2V_1)^{\frac{\frac{p_2+1}{q_2}}{2}} \quad (10)$$

When  $|s| < 1$

$$\dot{V}_1 = -\alpha_2(2V_1) - \beta_2(2V_1)^{\frac{p_2+1}{2}} \quad (11)$$

Obviously,  $\dot{V}_1 < 0$ , so the control system will stable in expect target non-singularity.

*Theorem 2*

The singularity item of the control input is restricted by the saturation function method, so that the system does not affect the stability analysis even if there is a singular region.

*Proof*

Defined inequality  $|\beta_1 \frac{p_1}{q_1} x_1^{\frac{p_1-1}{q_1}} x_2| > h$  as the singularity area.

The state variable  $x_1$  in first equation at system(3).

$$x_1(t) = x_1(0) + \int_0^t x_2(\tau) d\tau \quad (12)$$

When  $x_2(t) > 0$ ,  $x_1(t)$  will increase monotonically and

leave the singularity. If  $x_2(t) < 0$ ,  $x_1(t)$  will decrease monotonically and leave the singularity. Therefore, the existence of the singular region does not affect the results of the stability analysis.

*Arrival time analysis*

Consider the following differential equation

$$\dot{x} = -\alpha x^{\frac{1}{2} + \frac{m}{2n} + (\frac{m}{2n} - \frac{1}{2}) \text{sign}(|y|-1)} - \beta x^{\frac{p}{q}} \quad (13)$$

Where, assuming  $\alpha > 0, \beta > 0$ , and  $m, n, p, q$  is odd positive integers, the system will stable in a fix time.

*Proof*

The above system can be written as follows:

$$\begin{cases} \dot{x} = -\alpha x^{\frac{m}{2} - \beta x^{\frac{p}{q}}}, |x| > 1 \\ \dot{x} = -\alpha x - \beta x^{\frac{p}{q}}, |x| < 1 \end{cases} \quad (14)$$

A new variable is defined as  $z = x^{\frac{1-p}{q}}$ , so, the first equation in the above system can be written

$$\dot{z} + \frac{q-p}{q} \alpha z^{\frac{m-p}{n-q}} + \frac{q-p}{q} \beta = 0 \quad (15)$$

Let  $\varepsilon = [(m-n)q]/[n(q-p)]$ , can get

$$\dot{z} + \frac{q-p}{q} \alpha z^{1+\varepsilon} + \frac{q-p}{q} \beta = 0 \quad (16)$$

Similarly available, the second equation in system can be obtained

$$\dot{z} + \frac{q-p}{q} \alpha z + \frac{q-p}{q} \beta = 0 \quad (17)$$

So, the maximum convergence time is

$$\begin{aligned} \lim_{z_0 \rightarrow \infty} T(z_0) &= \lim_{z_0 \rightarrow \infty} \frac{q}{q-p} \left( \int_1^{z_0} \frac{1}{\alpha z^{1+\varepsilon} + \beta} dz + \int_0^1 \frac{1}{\alpha z + \beta} dz \right) \\ &< \frac{q}{q-p} \left( \frac{1}{\varepsilon \alpha} + \frac{1}{\alpha} \ln(1 + \frac{\alpha}{\beta}) \right) = \frac{1}{\alpha} \frac{n}{m-n} + \frac{q}{q-p} \frac{1}{\alpha} \ln(1 + \frac{\alpha}{\beta}) \end{aligned} \quad (18)$$

Without losing the general consideration of second order systems, the fix time for the controlled system to reach the slid surface is

$$T_1 < \frac{1}{2^{\frac{m_2+1}{2}} \alpha_2} \frac{2n_2}{m_2-n_2} + \frac{q_2}{q_2-p_2} \frac{1}{2\alpha_2} \ln(1 + \frac{2\alpha_2}{2^{\frac{p_2+1}{2}} \beta_2}) \quad (19)$$

When the controlled system reaches the sliding surface  $s=0$ , the target sliding mode of the system satisfies the following

$$\dot{x}_1 = x_2 = -\alpha_1 x_1^{\frac{1}{2} + \frac{m_1}{2n_1} + (\frac{m_1}{2n_1} - \frac{1}{2}) \text{sign}(|x_1|-1)} - \beta_1 x_1^{\frac{p_1}{q_1}} \quad (20)$$

The corresponding system  $x_1$  will converge in a fixed time:

$$T_2 < \frac{1}{2^{\frac{m_1+1}{2}} \alpha_1} \frac{2n_1}{m_1-n_1} + \frac{q_1}{q_1-p_1} \frac{1}{2\alpha_1} \ln(1 + \frac{2\alpha_1}{2^{\frac{p_1+1}{2}} \beta_1}) \quad (21)$$

The convergence time for system is

$$\begin{aligned} T = T_1 + T_2 &< \frac{1}{2^{\frac{m_2+1}{2}} \alpha_2} \frac{2n_2}{m_2-n_2} + \frac{q_2}{q_2-p_2} \frac{1}{2\alpha_2} \ln(1 + \frac{2\alpha_2}{2^{\frac{p_2+1}{2}} \beta_2}) \\ &+ \frac{1}{2^{\frac{m_1+1}{2}} \alpha_1} \frac{2n_1}{m_1-n_1} + \frac{q_1}{q_1-p_1} \frac{1}{2\alpha_1} \ln(1 + \frac{2\alpha_1}{2^{\frac{p_1+1}{2}} \beta_1}) \end{aligned} \quad (22)$$

#### IV. SIMULATION EXPERIMENT

The proposed control method is applied to suppress chaotic oscillation in studied power system. The parameters

of the controller is  $\alpha_1=10$  ,  $\beta_1=5$  ,  $\alpha_2=10$  ,  $\beta_2=5$  ,  
 $m_1=9$  ,  $n_1=5$  ,  $m_2=9$  ,  $n_2=5$  ,  $h=100$  ,  $p_1=5$  ,  $q_1=9$  ,  $p_2=5$  ,  
 $q_2=9$  . The initial value of the controlled system is

$[x_1, x_2] = [0.5, 0.1]$  .The dynamics of the system under this parameter have been obtained in the second part, and the system is in a chaotic state before it is controlled. As shown in figure 4, before being controlled, the system is in a chaos.

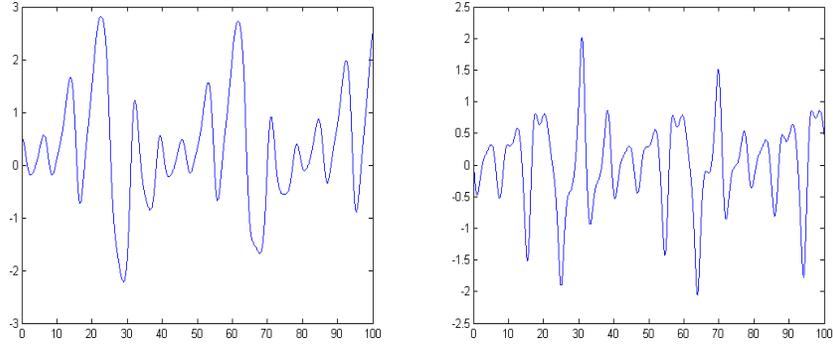


Figure 4. Time domain waveform(uncontrolled)

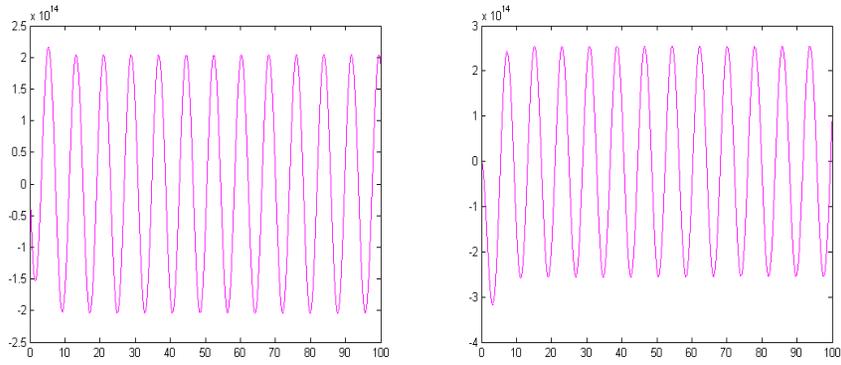


Figure 5. Time domain waveform after control

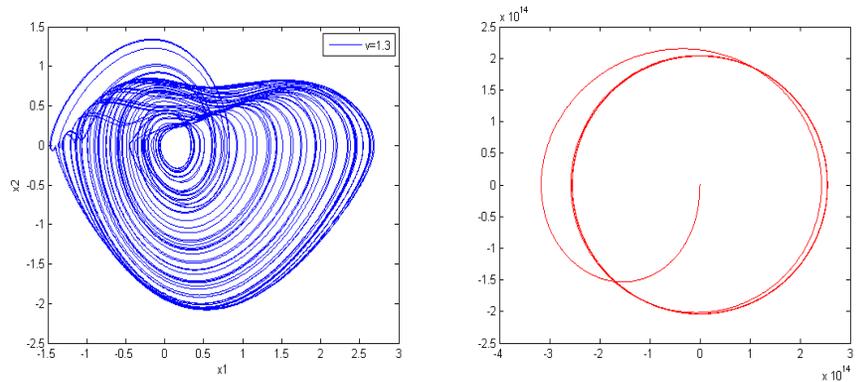


Figure 6. Phase plane plots

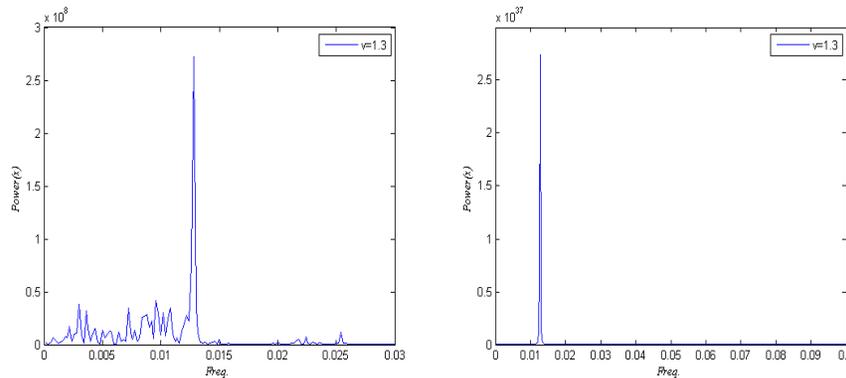


Figure 7. Power spectrum

From Fig.5, this paper can get that the system under chaotic oscillation is controlled by the control method proposed in this paper, then it will converge quickly to the desired target within a fixed time.

By contrasting the Fig.6, Fig.7, this can find this result. The system will be in chaos when it's uncontrolled, but the method which have proposed in this paper apply in the interconnected system, the system is stabilized.

#### V. CONCLUSION

In this paper, by plotting the Lyapunov exponent diagram, power spectrum and phase diagram of the interconnected power system, the influence of the amplitude of the electromagnetic disturbance for the system has been analyzed. According to the three stability criteria, when the system parameter  $\nu = 1.3$ , the interconnected power system will be in chaos. So, a non-singular terminal sliding mode control method with fixed time stability has been applied in the system when it's in chaos. By comparing the system output, we can find that the control method proposed in this paper can restrain the chaotic oscillation, and the interconnection system is stabilized in a fixed time. The singularity problem in the terminal sliding mode control is eliminated by introducing the saturation function. Due to the timing convergence characteristics and non-singularity of the proposed method, it will be applied to the actual power equipment.

#### VI. ACKNOWLEDGMENT

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