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NEW APPROACHES USING EXPONENTIAL TYPE ESTIMATOR WITH COST MODELLING FOR POPULATION MEAN ON SUCCESSIVE WAVES

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ABSTRACT

The key and fundamental purpose of sampling over successive waves lies in the varying nature of study character, it so may happen with ancillary information if the time lag between two successive waves is sufficiently large. Keeping the varying nature of auxiliary information in consideration, modern approaches have been proposed to estimate population mean over two successive waves. Four exponential ratio type estimators have been designed. The properties of proposed estimators have been elaborated theoretically including the optimum rotation rate.Cost models have also been worked out to minimize the total cost of the survey design over two successive waves. Dominances of the proposed estimators have been shown over well-known existing estimators. Simulation algorithms have been designed and applied to corroborate the theoretical results.

Key words: Successive sampling, Exponential type estimators, Dynamic ancillary information Population mean, Bias, Mean squared error, Optimum rotation rate.
Mathematics Subject Classification: 62D05.

1. Introduction

Real life facts always carry varyingnatures which are time dependent. In such circumstances where facts change over a period of time, one time enquiry may not serve the purpose of investigation since statistics observed previously contain superannuated information which may not be good enough to be used after a long period of time. Therefore surveys are being designed sophistically to make sure no possible error gets a margin to escape at least in terms of design. For this longitudinal surveys are considered to be best since in longitudinal surveys, facts are investigated more than once i.e. over the successive waves, Also a frame is

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provided for reducing the cost of survey by a partial replacement of sample units in sampling over successive waves.

Jessen (1942) is considered to be the pioneer for observing dynamics of facts over a long period of time through partial replacement of sample units over successive waves. The approach of sampling over successive waves has been made more fruitful by using twisted and novel ways to consider extra information along with the study character. Enhanced literature has been made availableby Patterson (1950), Narain (1953), Eckler (1955), Sen(1971, 1972, 1973), Gordon (1983), Singh et al. (1991), Arnab and Okafor (1992), Feng and Zou (1997), Biradar and Singh (2001), Singh and Singh (2001), Singh (2005), Singh and Priyanka (2006, 2007, 2008),Singh and Karna (2009), Singh and Prasad (2010), Singh et al. (2011), Singh et al. (2013), Bandyopadhyay and Singh(2014), Priyanka and Mittal (2014), Priyanka et al. (2015), Priyanka and Mittal (2015a, 2015b)etc.

It has been theoretically established that, in general, the linear regression estimator is more efficient than the ratio estimator except when the regression line y on x passes through the neighbourhood of the origin; in this case the efficiencies of these estimators are almost equal. Also in many practical situations where the regression line does not pass through the neighbourhood of the origin, in such cases the ratio estimator does not perform as good as the linear regression estimator. Here exponential type estimators play a vital role in increasing the precision of the estimates.Motivated with this idea we are aspired to develop unexampled estimators for estimating population mean over two successive waves applying the concept of exponential type ratio estimators. In this line of work, an attempt has been made to consider the dynamic nature of ancillary information also because as the time passes by, not only the nature of study variable changes but the nature of ancillary information also varies with respect to time in many real life phenomenon where time lag is very large between two successive waves.

For example, in a social survey one may desire to observe the number of females human trafficked from a particular region, the number of girls child birth may serve as ancillary information which is completely dynamic over a period of 8-10 years of time span. Similarly in a medicinal survey one may be interested to record the number of survivors from a cancerous disease, here the number of successfully tested drugs for the disease may not sustain to be stable over a period of 10 or 20 years. Likewise, in an economic survey the government may like to record the labor force, the total number of graduates in country may serve as an ancillary character to the study character but it surely inherent dynamic nature over a period of 5 or 10 years. Also in a tourism related survey, one may seek to record the total income (profit) from tourism in a particular country or state. In this kind of survey, total number of tourists visiting to the concerned place may be considered as the auxiliary information as communications and transportations services have emerged drastically to enhance the commutation of people from one place to another.

So such situations cannot be tackled considering the ancillary character to be stable since doing so will affect the final findings of the survey. Keeping the drawback of such flaws in consideration, this work deals in bringing modern approaches for estimating population mean over two successive waves.Four estimators have been habituated with a fine amalgamation of completely known dynamic ancillary information with exponential ratio type estimators.Their properties including optimum rotation rate and a model for optimum total cost have been proposed and discussed. Also detailed empirical illustrations have been done by doing a comparison of proposed estimators with well-known existing estimators in the literature of successive sampling. Simulation algorithms have been devised to make the proposed estimators work in practical environment efficiently.

2. Survey Design and Analysis

2.1. Sample Structure and Notations

Let $U = \left(U_1, U_2, \ldots, U_N\right)$ be the finite population of N units, which has been sampled over two successive waves. It is assumed that size of the population remains unchanged but values of units change over two successive waves. The character under study be denoted by x (y) on the first (second) waves respectively. It is assumed that information on an ancillary variable $z_1\left(z_2\right)$ dynamic in nature over the successive waves with completely known population mean $\bar{Z}_1\left(\bar{Z}_2\right)$ is readily available on both the successive waves and positively correlated to x and y respectively.Simple random sample (without replacement) of n units is taken at the first wave. A random subsample of $m = n\lambda$ units is retained for use at the second wave. Now at the current wave a simple random sample (without replacement) of u= (n-m) = nμ units is drawn afresh from the remaining (N-n) units of the population so that the sample size on the second wave remains the same. Let μ and $\lambda\left(\mu + \lambda = 1\right)$ are the fractions of fresh and matched samples respectively at the second (current) successive wave. The following notations are considered here after:

$\bar{X}, \bar{Y}, \bar{Z}_1, \bar{Z}_2$: Population means of the variables x, y, z_1 and z_2 respectively.

$\bar{y}_u, \bar{z}_u, \bar{x}_m, \bar{y}_m, \bar{z}_1\left(m\right), \bar{z}_2\left(m\right), \bar{x}_n, \bar{z}_1\left(n\right), \bar{z}_2\left(n\right)$: Sample mean of respective variate based on the sample sizes shown in suffice.

$\rho_{yx}, \rho_{xz_1}, \rho_{xz_2}, \rho_{yz_1}, \rho_{yz_2}, \rho_{z_1z_2}$: Correlation coefficient between the variables shown in suffices.

$S_x^2, S_y^2, S_{z_1}^2, S_{z_2}^2$: Population mean squared of variables x, y, z_1 and z_2 respectively.

2.2 Design of the Proposed Estimators $\check{\mathcal{T}}_{ij}\,(i, j=1, 2)$

For estimating the population mean $\bar{Y}$ at the current wave, two sets of estimators have been proposed. The first set of estimators is based on sample of size u drawn afresh at current occasion and is given by

$$\check{\mathcal{T}}_u = \{t_{1u}, t_{2u}\}, \tag{1}$$

where
$$t_{1u} = \bar{y}_u \left(\frac{\bar{Z}_2}{\bar{z}_2(u)}\right) \tag{2}$$

$$t_{2u} = \bar{y}_u \exp\left(\frac{\bar{Z}_2 - \bar{z}_2(u)}{\bar{Z}_2 + \bar{z}_2(u)}\right) \tag{3}$$

The second set of estimators is based on sample of size m common to both occasion and is

$$\check{\mathcal{T}}_m = \{t_{1m}, t_{2m}\}, \tag{4}$$

where
$$t_{1m} = \bar{y}_m \left(\frac{\bar{x}_n}{\bar{x}_m}\right) \exp\left(\frac{\bar{Z}_2 - \bar{z}_2(m)}{\bar{Z}_2 + \bar{z}_2(m)}\right) \tag{5}$$

$$t_{2m} = \bar{y}_m^* \left(\frac{\bar{x}_n^*}{\bar{x}_m^*}\right) \tag{6}$$

where $\bar{y}_m^* = \bar{y}_m \exp\left(\frac{\bar{Z}_2 - \bar{z}_2(m)}{\bar{Z}_2 + \bar{z}_2(m)}\right)$, $\bar{x}_m^* = \bar{x}_m \exp\left(\frac{\bar{Z}_1 - \bar{z}_1(m)}{\bar{Z}_1 + \bar{z}_1(m)}\right)$ and

$\bar{x}_n^* = \bar{x}_n \exp\left(\frac{\bar{Z}_1 - \bar{z}_1(n)}{\bar{Z}_1 + \bar{z}_1(n)}\right)$.

Hence, considering the convex combination of the two sets $\check{\mathcal{T}}_u$ and $\check{\mathcal{T}}_m$, we have the final estimators of the population mean $\bar{Y}$ on the current occasion as

$$\check{\mathcal{T}}_{ij} = \varpi_{ij} t_{iu} + (1-\varpi_{ij})\, t_{jm};\ (i, j=1, 2) \tag{7}$$

where $(t_{iu}, t_{jm}) \in \check{\mathcal{T}}_u \times \check{\mathcal{T}}_m$ and ϖ_{ij} are suitably chosen weights so as to minimize the mean squared error of the estimators $\check{\mathcal{T}}_{ij}\,(i, j=1, 2)$.

2.3. Analysis of the estimators $\check{\mathcal{T}}_{ij}\left(\text{i, j=1, 2}\right)$

2.3.1. Bias and Mean Squared Errors of the Proposed Estimators $\check{\mathcal{T}}_{ij}\left(\text{i, j=1, 2}\right)$

The properties of the proposed estimators $\check{\mathcal{T}}_{ij}\left(\text{i, j=1, 2}\right)$ are derived under the following large sample approximations

$\bar{y}_u = \bar{Y}\left(1 + e_0\right),\ \bar{y}_m = \bar{Y}\left(1 + e_1\right),\ \bar{x}_m = \bar{X}\left(1 + e_2\right), \bar{x}_n = \bar{X}\left(1 + e_3\right), \bar{z}_2\left(u\right) = \bar{Z}_2\left(1 + e_4\right),$
$\bar{z}_2\left(m\right) = \bar{Z}_2\left(1 + e_5\right), \bar{z}_1\left(m\right) = \bar{Z}_1\left(1 + e_6\right)$ and $\bar{z}_1\left(n\right) = \bar{Z}_1\left(1 + e_7\right)$ such that $|e_i| < 1\ \forall\ i = 0,...,7.$

The estimators belonging to the sets $\check{\mathcal{T}}_u$ and $\check{\mathcal{T}}_m\left(\text{i, j=1, 2}\right)$ are ratio, exponential ratio, ratio to exponential ratio and chain type ratio to exponential ratio type in nature respectively. Hence they are biased for population mean $\bar{Y}$. Therefore, the final estimators $\check{\mathcal{T}}_{ij}\left(\text{i, j=1, 2}\right)$ defined in equation (7) are also biased estimators of $\bar{Y}$. The bias $B\left(.\right)$ and mean squared errors $M\left(.\right)$ of the proposed estimators $\check{\mathcal{T}}_{ij}\left(\text{i, j=1, 2}\right)$ are obtained up to first order of approximations and thus we have following theorems:

Theorem 2.3.1. Bias of the estimators $\check{\mathcal{T}}_{ij}\left(\text{i, j=1, 2}\right)$ to the first order of approximations are obtained as

$$B\left(\check{\mathcal{T}}_{ij}\right) = \varpi_{ij}\, B\left(t_{iu}\right) + \left(1 - \varpi_{ij}\right) B\left(t_{jm}\right);\ (\text{i, j=1,2}), \tag{8}$$

where
$$B\left(t_{1u}\right) = \frac{1}{u}\bar{Y}\left(\frac{C_{0002}}{\bar{Z}_2^2} - \frac{C_{0101}}{\bar{Y}\,\bar{Z}_2}\right), \tag{9}$$

$$B\left(t_{2u}\right) = \frac{1}{u}\bar{Y}\left(\frac{3}{8}\frac{C_{0002}}{\bar{Z}_2^2} - \frac{1}{2}\frac{C_{0101}}{\bar{Y}\,\bar{Z}_2}\right), \tag{10}$$

$$B\left(t_{1m}\right) = \bar{Y}\left(\frac{1}{m}\left(\frac{C_{2000}}{\bar{X}^2} + \frac{3}{8}\frac{C_{0002}}{\bar{Z}_2^2} - \frac{C_{1100}}{\bar{X}\bar{Y}} - \frac{1}{2}\frac{C_{0101}}{\bar{Y}\bar{Z}_2} + \frac{1}{2}\frac{C_{1001}}{\bar{X}\bar{Z}_2}\right) + \frac{1}{n}\left(\frac{C_{1100}}{\bar{X}\bar{Y}} - \frac{C_{2000}}{\bar{X}^2} - \frac{1}{2}\frac{C_{1001}}{\bar{X}\bar{Z}_2}\right)\right), \tag{11}$$

and

$$B\left(t_{2m}\right) = \bar{Y}\left(\frac{1}{m}\left(\frac{C_{2000}}{\bar{X}^2} - \frac{1}{8}\frac{C_{0020}}{\bar{Z}_1^2} + \frac{3}{8}\frac{C_{0002}}{\bar{Z}_2^2} - \frac{C_{1100}}{\bar{X}\bar{Y}} - \frac{1}{2}\frac{C_{1010}}{\bar{X}\bar{Z}_1} + \frac{1}{2}\frac{C_{1001}}{\bar{X}\bar{Z}_2} + \frac{1}{2}\frac{C_{0110}}{\bar{Y}\bar{Z}_1} - \frac{1}{2}\frac{C_{0101}}{\bar{Y}\bar{Z}_2} - \frac{1}{4}\frac{C_{0011}}{\bar{Z}_1\bar{Z}_2}\right) + \frac{1}{n}\left(\frac{1}{8}\frac{C_{002}}{\bar{Z}_1^2} - \frac{C_{2000}}{\bar{X}^2} + \frac{C_{1100}}{\bar{X}\bar{Y}} + \frac{1}{2}\frac{C_{1010}}{\bar{X}\bar{Z}_1} - \frac{1}{2}\frac{C_{1001}}{\bar{X}\bar{Z}_2} - \frac{1}{2}\frac{C_{0101}}{\bar{Y}\bar{Z}_2} + \frac{1}{4}\frac{C_{0011}}{\bar{Z}_1\bar{Z}_2}\right)\right) \tag{12}$$

where $C_{rstq} = E\left[\left(x_i - \bar{X}\right)^r \left(y_i - \bar{Y}\right)^s \left(z_{1i} - \bar{Z}_1\right)^t \left(z_{2i} - \bar{Z}_2\right)^q\right]; \left(r, s, t, q\right) \geq 0$.

Theorem 2.3.2. Mean squared errors of the estimators $\check{\mathcal{T}}_{ij}\left(i, j=1, 2\right)$ to the first order of approximations are obtained as

$$M\left(\check{\mathcal{T}}_{ij}\right) = \varpi_{ij}^2\, M\left(t_{iu}\right) + \left(1 - \varpi_{ij}\right)^2 M\left(t_{jm}\right) + 2\, \varpi_{ij}\left(1 - \varpi_{ij}\right) Cov\left(t_{iu}, t_{jm}\right); \quad (i,j=1,2) \tag{13}$$

where $$M\left(t_{1u}\right) = \frac{1}{u} A_1\, S_y^2 \tag{14}$$

$$M\left(t_{2u}\right) = \frac{1}{u} A_2\, S_y^2 \tag{15}$$

$$M\left(t_{1m}\right) = \left(\frac{1}{m} A_3 + \frac{1}{n} A_4\right) S_y^2 \tag{16}$$

$$M\left(t_{2m}\right) = \left(\frac{1}{m} A_5 + \frac{1}{n} A_6\right) S_y^2 \tag{17}$$

$Cov\left(t_{iu}, t_{jm}\right)=0$, $A_1 = 2\left(1 - \rho_{yz_2}\right)$, $A_2 = \frac{5}{4} - \rho_{yz_2}$, $A_3 = \frac{9}{4} - 2\rho_{yx} - \rho_{yz_2} + \rho_{xz_2}$,

$A_4 = 2\rho_{yx} - \rho_{xz_2} - 1$,

$A_5 = \frac{5}{2} - 2\rho_{yx} - \rho_{xz_1} + \rho_{xz_2} + \rho_{yz_1} - \rho_{yz_2} - \frac{1}{2}\rho_{z_1z_2}$ and $A_6 = 2\rho_{yx} + \rho_{xz_1} - \rho_{xz_2} - \rho_{yz_1} + \frac{1}{2}\rho_{z_1z_2} - \frac{5}{4}$.

2.3.2. Minimum Mean Squared Errors of the Proposed Estimators $\check{\mathcal{T}}_{ij}\left(i, j=1, 2\right)$

Since the mean squared errors of the estimators $\check{\mathcal{T}}_{ij}\left(i, j=1, 2\right)$ given in equation (13) are the functions of unknown constants $\varpi_{ij}\left(i, j = 1, 2\right)$, therefore, they are minimized with respect to ϖ_{ij} and subsequently the optimum values of $\varpi_{ij}\left(i, j = 1, 2\right)$ and $M\left(\check{\mathcal{T}}_{ij}\right)_{opt.}\left(i, j=1, 2\right)$ are obtained as

$$\varpi_{ij_{opt.}} = \frac{M\left(t_{jm}\right)}{M\left(t_{iu}\right) + M\left(t_{jm}\right)}; (i, j = 1, 2) \tag{18}$$

$$M\left(\check{\mathcal{T}}_{ij}\right)_{opt.} = \frac{M\left(t_{iu}\right) . M\left(t_{jm}\right)}{M\left(t_{iu}\right) + M\left(t_{jm}\right)}; \quad \left(i, j = 1, 2\right) \tag{19}$$

Further, substituting the values of the mean squared errors of the estimators defined in equations (14)-(17) in equation (18)-(19), the simplified values of $\varpi_{ij_{opt.}}$ and $M(\check{\mathcal{T}}_{ij})_{opt.}$ are obtained as

$$\varpi_{11_{opt.}} = \frac{\mu_{11}\left[\mu_{11} A_4 - (A_3 + A_4)\right]}{\left[\mu_{11}^2 A_4 - \mu_{11}(A_3 + A_4 - A_1) - A_1\right]} \tag{20}$$

$$\varpi_{12_{opt.}} = \frac{\mu_{12}\left[\mu_{12} A_6 - (A_5 + A_6)\right]}{\left[\mu_{12}^2 A_6 - \mu_{12}(A_5 + A_6 - A_1) - A_1\right]} \tag{21}$$

$$\varpi_{21_{opt.}} = \frac{\mu_{21}\left[\mu_{21} A_4 - (A_3 + A_4)\right]}{\left[\mu_{21}^2 A_4 - \mu_{21}(A_3 + A_4 - A_2) - A_2\right]} \tag{22}$$

$$\varpi_{22_{opt.}} = \frac{\mu_{22}\left[\mu_{22} A_6 - (A_5 + A_6)\right]}{\left[\mu_{22}^2 A_6 - \mu_{22}(A_5 + A_6 - A_2) - A_2\right]} \tag{23}$$

$$M(\check{\mathcal{T}}_{11})_{opt.} = \frac{1}{n}\frac{\left[\mu_{11} B_1 - B_2\right]S_y^2}{\left[\mu_{11}^2 A_4 - \mu_{11} B_3 - A_1\right]} \tag{24}$$

$$M(\check{\mathcal{T}}_{12})_{opt.} = \frac{1}{n}\frac{\left[\mu_{12} B_4 - B_5\right]S_y^2}{\left[\mu_{12}^2 A_6 - \mu_{12} B_6 - A_1\right]} \tag{25}$$

$$M(\check{\mathcal{T}}_{21})_{opt.} = \frac{1}{n}\frac{\left[\mu_{21} B_7 - B_8\right]S_y^2}{\left[\mu_{21}^2 A_4 - \mu_{21} B_9 - A_2\right]} \tag{26}$$

$$M(\check{\mathcal{T}}_{22})_{opt.} = \frac{1}{n}\frac{\left[\mu_{22} B_{10} - B_{11}\right]S_y^2}{\left[\mu_{22}^2 A_6 - \mu_{22} B_{12} - A_2\right]} \tag{27}$$

where
$B_1 = A_1A_4$, $B_2 = A_1A_3 + A_1A_4$, $B_3 = A_3 + A_4 - A_1$, $B_4 = A_1A_6$, $B_5 = A_1A_5 + A_1A_6$, $B_6 = A_5 + A_6 - A_1$, $B_7 = A_2A_4$, $B_8 = A_2A_3 + A_2A_4$, $B_9 = A_3 + A_4 - A_2$, $B_{10} = A_2A_6$ $B_{11} = A_2A_5 + A_2A_6$, $B_{12} = A_5 + A_6 - A_2$ and $\mu_{ij}(i, j = 1, 2)$ are the fractions of the sample drawn afresh at the current(second) wave.

2.3.3. Optimum Rotation Rate for the Estimators $\check{\mathcal{T}}_{ij}(i, j=1, 2)$

Since the mean squared errors of the proposed estimators $\check{\mathcal{T}}_{ij}(i, j=1, 2)$ are the function of the $\mu_{ij}(i, j = 1, 2)$, hence to estimate population mean $\bar{Y}$ with maximum precision and minimum cost,an amicable fraction of sample drawn afresh is required at the current wave. For this the mean squared errors of the estimators $\check{\mathcal{T}}_{ij}\left(i, j{=}1, 2\right)$ in equations (24) – (27) have been minimized with respect to $\mu_{ij}\left(i, j = 1, 2\right)$. Hence an optimum rotation rate has been obtained for each of the estimators $\check{\mathcal{T}}_{ij}\left(i, j{=}1, 2\right)$ and given as:

$$\mu_{11} = \frac{C_2 \pm \sqrt{C_2^2 - C_1 C_3}}{C_1} \tag{28}$$

$$\mu_{12} = \frac{C_5 \pm \sqrt{C_5^2 - C_4 C_6}}{C_4} \tag{29}$$

$$\mu_{21} = \frac{C_8 \pm \sqrt{C_8^2 - C_7 C_9}}{C_7} \tag{30}$$

$$\mu_{22} = \frac{C_{11} \pm \sqrt{C_{11}^2 - C_{10} C_{12}}}{C_{10}} \tag{31}$$

where
$C_1 = A_4B_1,\ C_2 = A_4B_2,\ C_3 = A_1B_1 + B_2B_3,\ C_4 = A_6B_4,\ C_5 = A_6B_5,\ C_6 = A_1B_4 + B_5B_6$
$C_7 = A_4B_7,\ C_8 = A_4B_8,\ C_9 = A_2B_7 + B_8B_9,\ C_{10} = A_6B_{10},\ C_{11} = A_6B_{11}$ and $C_{12} = A_2B_{10} + B_{11}B_{12}$.

The real values of $\mu_{ij}\left(i, j = 1, 2\right)$ exist, iff $C_2^2 - C_1 C_3 \geq 0$, $C_5^2 - C_4 C_6 \geq 0$, $C_8^2 - C_7 C_9 \geq 0$, and $C_{11}^2 - C_{10} C_{12} \geq 0$ respectively. For any situation, which satisfies these conditions, two real values of $\mu_{ij}\left(i, j = 1, 2\right)$ may be possible , hence to choose a value of $\mu_{ij}\left(i, j = 1, 2\right)$, it should be taken care of that $0 \leq \mu_{ij} \leq 1$, all other values of $\mu_{ij}\left(i, j = 1, 2\right)$ are inadmissible. If both the real values of $\mu_{ij}\left(i, j = 1, 2\right)$ are admissible, the lowest one will be the best choice as it reduces the total cost of the survey. Substituting the admissible value of μ_{ij} say $\mu_{ij}^{(0)}\left(i, j = 1, 2\right)$ from equation (28) - (31) in equation (24) - (27) respectively , we get the optimum values of the mean squared errors of the estimators $\check{\mathcal{T}}_{ij}\left(i, j = 1, 2\right)$ with respect to ϖ_{ij} as well as $\mu_{ij}\left(i, j = 1, 2\right)$ which are given as

$$M\left(\check{\mathcal{T}}_{11}\right)^{*}_{\text{opt.}} = \frac{\left[\mu_{11}^{(0)} B_1 - B_2\right] S_y^2}{n\left[\mu_{11}^{(0)\,2} A_4 - \mu_{11}^{(0)} B_3 - A_1\right]} \tag{32}$$

$$M\left(\check{\mathcal{T}}_{12}\right)^{*}_{opt.} = \frac{\left[\mu_{12}^{(0)}\ B_4\ - B_5\right]\ S_y^2}{n\left[\mu_{12}^{(0)\,2}\ A_6\ - \mu_{12}^{(0)}\ B_6\ - A_1\right]} \tag{33}$$

$$M\left(\check{\mathcal{T}}_{21}\right)^{*}_{opt.} = \frac{\left[\mu_{21}^{(0)}\ B_7\ - B_8\right]\ S_y^2}{n\left[\mu_{21}^{(0)\,2}\ A_4\ - \mu_{21}^{(0)}\ B_9\ - A_2\right]} \tag{34}$$

$$M\left(\check{\mathcal{T}}_{22}\right)^{*}_{opt.} = \frac{\left[\mu_{22}^{(0)}\ B_{10}\ - B_{11}\right]\ S_y^2}{n\left[\mu_{22}^{(0)\,2}\ A_6\ - \mu_{22}^{(0)}\ B_{12}\ - A_2\right]} \tag{35}$$

3. Cost Analysis

The total cost of survey design and analysis over two successive waves is modelled as:

$$C_T = nc_f + mc_r + uc_s \tag{36}$$

where c_f : The average per unit cost of investigating and processing data at previous (first)wave,

c_r : The average per unit cost of investigating and processing retained data at current wave,

c_s : The average per unit cost of investigating and processing freshly drawndata at current wave.

Remark 3.1: $c_f < c_r < c_s$, when there is a large gap between two successive waves, the cost of investigating a single unit involved in the survey sample should be greater than before (at previous occasion) since as time passes by different commodities (software) and services (human resources, daily wages and conveyance) become expensive so the cost incurring at second occasion increases in a considerable amount. Also the average cost of investigating a retained unit from previous wave should be lesser than investigating a freshly drawn sample unit since survey investigator as well as respondent has some experiences from the previous wave.

Theorem 3.1.1: The optimum total cost for the proposed estimators $\check{\mathcal{T}}_{ij}\left(i, j=1, 2\right)$ is derived as

$$C_T\left(\check{\mathcal{T}}_{ij}\right)_{opt.} = n\left\{c_f + c_s + \left(1 - \mu_{ij}^{(0)}\right)\left(c_r - c_s\right)\right\} \forall\ i, j=1, 2 \tag{37}$$

Remark 3.2: The optimum total costsobtained in equation (37) are dependent on the value of n. Therefore, if a suitable guess of n is available, it can be used for obtaining optimum total cost of the survey by above equation. However, in the absence of suitable guess of n, it may be estimated by following Cochran (1977).

4. Efficiency Comparison

To evaluate the performance of the proposed estimators, the estimators $\check{\mathcal{T}}_{ij}\left(i, j=1, 2\right)$ at optimum conditions, are compared with the sample mean estimator $\bar{y}_n$, when there is no matching from previous wave and the estimator $\hat{\bar{Y}}$ due to Jessen (1942) given by

$$\hat{\bar{Y}} = \psi\, \bar{y}_u + \left(1 - \psi\right) \bar{y}_m^{'} , \tag{38}$$

where $\bar{y}_m^{'} = \bar{y}_m + \beta_{yx}\left(\bar{x}_n - \bar{x}_m\right)$, β_{yx} is the population regression coefficient of y on x and ψ is an unknown constant to be determined so as to minimize the mean squared error of the estimator $\hat{\bar{Y}}$. The estimators $\bar{y}_n$ and $\hat{\bar{Y}}$ are unbiased for population mean, therefore variance of the estimators $\bar{y}_n$ and $\hat{\bar{Y}}$ at optimum conditions are given as

$$V\left(\bar{y}_n\right) = \frac{1}{n} S_y^2 , \tag{39}$$

$$V\left(\hat{\bar{Y}}\right)_{opt.}^{*} = \left(\frac{1}{2}\left(1 + \sqrt{1 - \rho_{yx}^2}\right)\right)\frac{S_y^2}{n} , \tag{40}$$

and the fraction of sample to be drawn afresh for the estimator $\hat{\bar{Y}}$

$$\mu_J = \frac{1}{1 + \sqrt{1 - \rho_{yx}^2}} \tag{41}$$

The percent relative efficiencies $E_{ij}(M)$ and $E_{ij}(J)$ of the estimator $\check{\mathcal{T}}_{ij}\left(i, j=1, 2\right)$ (under optimum conditions) with respect to $\bar{y}_n$ and $\hat{\bar{Y}}$ are respectively given by

$$E_{ij}(M) = \frac{V\left(\bar{y}_n\right)}{M\left(\check{\mathcal{T}}_{ij}\right)_{opt.}^{*}} \times 100 \text{ and } E_{ij}(J) = \frac{V\left(\hat{\bar{Y}}\right)_{opt.}^{*}}{M\left(\check{\mathcal{T}}_{ij}\right)_{opt.}^{*}} \times 100 \, (i, j=1, 2). \tag{42}$$

5. Numerical Illustrations and Simulation

5.1. Generalization of empirical study

A generalized study has been done to show the impact of varying ancillary information in enhancing the performance of the proposed estimators $\check{\mathcal{T}}_{ij}\,(i, j{=}1, 2)$. To elaborate the scenario, various choices of correlation coefficients of study and auxiliary variables have been considered. The results obtained have been shown in Table 1.

Table 1. Generalized empirical results while the proposed estimators $\check{\mathcal{T}}_{ij}\,(i, j{=}1, 2)$ have been compared to the estimators $\bar{y}_n$ and $\hat{\bar{Y}}$ for $\rho_{yz_1} = \rho_{yz_2} = \rho_1$ and $\rho_{xz_1} = \rho_{xz_2} = \rho_2$.

$\rho_{z_1z_2} = \rho_{yx} = 0.5$													
ρ_2	ρ_1	μ_J	$\mu_{11}^{(0)}$	$\mu_{12}^{(0)}$	$\mu_{21}^{(0)}$	$\mu_{22}^{(0)}$	$E_{11}(M)$	$E_{12}(M)$	$E_{21}(M)$	$E_{22}(M)$	$E_{11}(j)$	$E_{12}(j)$	$E_{21}(j)$	$E_{22}(j)$
0.4	**0.6**	0.53	0.66	0.58	0.44	0.41	119.69	114.58	135.48	128.91	111.67	106.90	126.41	120.28
	0.8	0.53	0.33	0.32	0.42	0.37	197.61	176.31	187.18	166.66	184.38	164.50	174.64	155.50
0.5	**0.6**	0.53	0.61	0.58	0.42	0.41	117.08	114.58	132.04	128.91	109.24	106.90	123.20	120.28
	0.8	0.53	0.33	0.32	0.40	0.37	191.44	176.31	181.18	166.66	178.61	164.50	169.04	155.50
0.6	**0.6**	0.53	0.58	0.58	0.41	0.41	114.58	114.58	128.91	128.91	106.90	106.99	120.28	120.28
	0.8	0.53	0.33	0.32	0.39	0.37	185.89	176.31	175.84	166.66	173.44	164.50	164.06	155.50
0.7	**0.6**	0.53	0.55	0.58	0.40	0.41	112.22	114.58	126.04	128.91	104.70	106.90	117.60	120.28
	0.8	0.53	0.32	0.32	0.38	0.37	180.88	176.31	171.03	166.66	168.76	164.50	159.57	155.50
$\rho_{z_1z_2} = \rho_{yx} = 0.6$														
ρ_2	ρ_1	μ_J	$\mu_{11}^{(0)}$	$\mu_{12}^{(0)}$	$\mu_{21}^{(0)}$	$\mu_{22}^{(0)}$	$E_{11}(M)$	$E_{12}(M)$	$E_{21}(M)$	$E_{22}(M)$	$E_{11}(j)$	$E_{12}(j)$	$E_{21}(j)$	$E_{22}(j)$
0.4	**0.6**	0.55	0.87	0.69	0.46	0.44	124.52	121.01	143.54	137.34	112.07	108.91	129.18	123.60
	0.8	0.55	0.29	0.33	0.45	0.40	212.25	188.59	201.85	178.44	191.02	169.73	181.58	160.54
0.5	**0.6**	0.55	0.73	0.69	0.45	0.44	122.31	121.01	139.24	137.34	110.08	108.91	125.36	123.60
	0.8	0.55	0.32	0.33	0.43	0.40	204.53	188.59	193.99	178.44	184.08	169.73	174.59	160.59
0.6	**0.6**	0.55	0.66	0.69	0.44	0.44	119.69	121.01	135.48	137.34	107.72	108.91	121.94	123.60
	0.8	0.55	0.33	0.33	0.42	0.40	197.61	188.59	187.18	178.44	177.85	169.73	168.46	160.54
0.7	**0.6**	0.55	0.61	0.69	0.42	0.44	117.08	121.01	132.04	137.34	105.37	108.91	118.84	123.60
	0.8	0.55	0.33	0.33	0.40	0.40	191.44	188.59	181.18	178.44	172.29	169.73	163.06	160.59

Note: The values for $\mu_j, \mu_{11}^{(0)}, \mu_{12}^{(0)}, \mu_{21}^{(0)}$ and $\mu_{22}^{(0)}$ have been rounded off up to two places of decimal for presentation.

5.2. Generalized study based on correlation coefficients andoptimum total cost model

To validate the proposed cost model, a hypothetical survey design has been assumed in which various choices of correlation coefficient and different input costs have been considered over two successive waves.

Table 2. Optimum total cost of the survey design at the current wave of the proposed estimators $\check{\mathcal{T}}_{ij}$ $(i, j=1, 2)$

$\rho_{yx}=0.5$, $n=30$, C_f = ₹ 50.00, C_r = ₹ 75.00 and C_s = ₹ 80.00

ρ_2	ρ_1	μ_J	$\mu_{11}^{(0)}$	$\mu_{12}^{(0)}$	$\mu_{21}^{(0)}$	$\mu_{22}^{(0)}$	$C_T(J)$	$C_T(11)$	$C_T(12)$	$C_T(21)$	$C_T(22)$
0.5	**0.6**	0.53	0.61	0.58	0.42	0.41	3830.4	3842.7	3837.5	3814.4	3812.8
	0.8	0.53	0.33	0.32	0.40	0.37	3830.4	3799.9	3798.2	3811.2	3806.3
0.6	**0.6**	0.53	0.58	0.58	0.41	0.41	3830.4	3837.5	3837.5	3812.8	3812.8
	0.8	0.53	0.33	0.32	0.39	0.37	3830.4	3799.5	3798.2	3809.3	3806.3
0.7	**0.6**	0.53	0.55	0.58	0.40	0.41	3830.4	3833.4	3837.5	3811.4	3812.8
	0.8	0.53	0.32	0.32	0.38	0.37	3830.4	3798.9	3798.2	3807.7	3806.3

$\rho_{yx}=0.6$, $n=30$, C_f = ₹ 50.00, C_r = ₹ 75.00 and C_s = ₹ 80.00

ρ_2	ρ_1	μ_J	$\mu_{11}^{(0)}$	$\mu_{12}^{(0)}$	$\mu_{21}^{(0)}$	$\mu_{22}^{(0)}$	$C_T(J)$	$C_T(11)$	$C_T(12)$	$C_T(21)$	$C_T(22)$
0.5	**0.6**	0.55	0.73	0.69	0.45	0.44	3833.3	3860.9	3854.8	3817.9	3817.0
	0.8	0.55	0.32	0.33	0.43	0.40	3833.3	3798.9	3799.8	3815.5	3810.2
0.6	**0.6**	0.55	0.66	0.69	0.44	0.44	3833.3	3849.9	3854.4	3816.1	3817.0
	0.8	0.55	0.33	0.33	0.42	0.40	3833.3	3833.3	3799.8	3813.2	3810.2
0.7	**0.6**	0.55	0.61	0.69	0.42	0.44	3833.3	3842.7	3854.8	3814.4	3817.0
	0.8	0.55	0.33	0.33	0.40	0.40	3833.3	3833.3	3799.9	3811.2	3810.2

$\rho_{yx}=0.5$, $n=40$, C_f = ₹ 50.00, C_r = ₹ 75.00 and C_s = ₹ 80.00

ρ_2	ρ_1	μ_J	$\mu_{11}^{(0)}$	$\mu_{12}^{(0)}$	$\mu_{21}^{(0)}$	$\mu_{22}^{(0)}$	$C_T(J)$	$C_T(11)$	$C_T(12)$	$C_T(21)$	$C_T(22)$
0.5	**0.6**	0.53	0.61	0.58	0.42	0.41	5107.2	5123.7	5116.7	5085.8	5083.8
	0.8	0.53	0.33	0.32	0.40	0.37	5107.2	5066.6	5064.3	5081.5	5075.0
0.6	**0.6**	0.53	0.58	0.58	0.41	0.41	5107.2	5116.7	5116.7	5083.8	5083.8
	0.8	0.53	0.33	0.32	0.39	0.37	5107.2	5066.0	5064.3	5079.1	5075.0
0.7	**0.6**	0.53	0.55	0.58	0.40	0.41	5107.2	5111.2	5116.7	5081.9	5083.8
	0.8	0.53	0.32	0.32	0.38	0.37	5107.2	5062.2	5064.3	5077.0	5075.0

$\rho_{yx}=0.6$, $n=40$, C_f = ₹ 50.00, C_r = ₹ 75.00 and C_s = ₹ 80.00

ρ_2	ρ_1	μ_J	$\mu_{11}^{(0)}$	$\mu_{12}^{(0)}$	$\mu_{21}^{(0)}$	$\mu_{22}^{(0)}$	$C_T(J)$	$C_T(11)$	$C_T(12)$	$C_T(21)$	$C_T(22)$
0.5	**0.6**	0.55	0.73	0.69	0.45	0.44	5111.1	5147.9	5139.7	5090.5	5089.3
	0.8	0.55	0.32	0.33	0.43	0.40	5111.1	5065.1	5066.3	5087.3	5080.3
0.6	**0.6**	0.55	0.66	0.69	0.44	0.44	5111.1	5133.3	5139.7	5088.1	5089.3
	0.8	0.55	0.33	0.33	0.42	0.40	5111.1	5066.5	5066.3	5084.2	5080.3
0.7	**0.6**	0.55	0.61	0.69	0.42	0.44	5111.1	5123.7	5139.7	5085.8	5089.3
	0.8	0.55	0.33	0.33	0.40	0.40	5111.1	5066.6	5066.3	5081.5	5080.3

5.3. Monte Carlo Simulation

Monte Carlo simulation has been performed to get an overview of the proposed estimators in practical scenario through considering different choices of n and μ for better analysis.

Following set has been considered for the simulation study

Set I: $n = 20$, $\mu = 0.15$, $(m = 17, u = 3)$.

5.3.1. Simulation Algorithm

(i) Choose 5000 samples of size n=20 using simple random sampling without replacement on first wave for both the study and auxiliary variable.

(ii) Calculate sample mean $\bar{x}_{n|k}$ and $\bar{z}_{1|k}(n)$ for k =1, 2, - - -, 5000.

(iii) Retain m=17 units out of each $n=20$ sample units of the study and auxiliary variables at the first wave.

(iv) Calculate sample mean $\bar{x}_{m|k}$ and $\bar{z}_{1|k}(m)$ for k= 1, 2, - - -, 5000.

(v) Select u=3 units using simple random sampling without replacement from N-n=31 units of the population for study and auxiliary variables at second (current) wave.

(vi) Calculate sample mean $\bar{y}_{u|k}$ and $\bar{z}_{2|k}(m)$ for k = 1, 2, - - -, 5000.

(vii) Iterate the parameter ϖ from 0.1 to 0.9 with a step of 0.2.

(viii) Iterate ψ from 0.1 to 0.9 with a step of 0.1 within (ix).

(ix) Calculate the percent relative efficiencies of the proposed estimator $\check{\mathcal{T}}_{ij}\,(i, j=1, 2)$ with respect to estimator to $\bar{y}_n$ and $\hat{\hat{\bar{Y}}}$ as

$$E\left(\check{\mathcal{T}}_{ij}, M\right) = \frac{\sum_{k=1}^{5000}\left[\check{\mathcal{T}}_{ij|k} - \bar{y}_{n|k}\right]^2}{\sum_{k=1}^{5000}\left[\check{\mathcal{T}}_{ij|k}\right]^2} \times 100 \text{ and } E\left(\check{\mathcal{T}}_{ij}, J\right) = \frac{\sum_{k=1}^{5000}\left[\check{\mathcal{T}}_{ij|k} - \hat{\hat{\bar{Y}}}_{|k}\right]^2}{\sum_{k=1}^{5000}\left[\check{\mathcal{T}}_{ij|k}\right]^2} \times 100 \;;(i, j=1, 2),\ k=1, 2, ..., 5000.$$

Table 3. Simulation Results when proposed estimator $\check{\mathcal{T}}_{ij}\left(\mathrm{i, j=1, 2}\right)$ have been compared to $\overline{y}_n$

SET \ ϖ_{ij}		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
I	$E\left(\check{\mathcal{T}}_{11}, M\right)$	218.44	244.19	271.66	303.33	333.45	364.67	393.36	419.85	439.71
	$E\left(\check{\mathcal{T}}_{12}, M\right)$	461.69	514.46	566.60	619.47	665.56	703.63	731.26	746.47	743.76
	$E\left(\check{\mathcal{T}}_{21}, M\right)$	231.69	260.16	283.97	304.00	306.75	299.67	281.22	256.46	228.66
	$E\left(\check{\mathcal{T}}_{22}, M\right)$	505.81	562.87	585.67	578.89	529.97	467.34	397.98	334.78	280.42

Table 4. Simulation results when the proposed estimator $\check{\mathcal{T}}_{11}$ is compared with the estimator $\hat{\overline{Y}}$

ψ \ ϖ_{11}	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
0.1	182.55	153.16	137.82	159.12	205.37	293.91	390.68	499.10	660.27
0.2	209.72	166.19	153.72	179.11	229.31	301.97	423.52	550.37	732.32
0.3	229.76	184.83	170.77	196.28	255.02	336.89	470.10	618.90	818.38
0.4	252.68	205.47	188.91	216.26	278.49	376.42	523.01	679.76	908.90
0.5	278.45	227.24	209.50	239.44	304.72	411.04	574.21	748.40	994.88
0.6	303.72	249.08	229.98	261.05	333.80	449.52	625.62	813.86	1085.2
0.7	327.71	270.01	249.29	281.45	362.61	482.77	674.92	882.95	1164.0
0.8	350.68	287.18	267.02	300.12	385.81	515.96	718.68	947.91	1240.0
0.9	366.09	300.07	280.72	315.76	404.82	541.84	752.31	995.79	1298.8

Table 5. Simulation results when the proposed estimator $\check{\mathcal{T}}_{12}$ is compared with the estimator $\hat{\overline{Y}}$

ψ \ ϖ_{12}	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
0.1	427.90	343.13	312.59	375.43	472.41	634.04	887.31	1166.2	1504.7
0.2	475.65	373.99	347.93	407.95	521.89	684.91	962.09	1278.4	1656.4
0.3	516.76	407.22	383.11	444.72	572.07	757.85	1047.3	1398.8	1833.6
0.4	556.92	445.77	416.14	480.80	614.70	829.45	1138.9	1507.8	1992.2
0.5	596.58	480.22	449.92	516.79	655.86	884.66	1221.4	1616.5	2127.7
0.6	627.38	510.46	477.32	544.38	696.06	935.55	1286.6	1703.9	2248.6
0.7	650.44	531.63	496.40	562.32	724.39	961.78	1335.1	1767.5	2313.1
0.8	663.15	538.25	507.13	569.47	733.17	977.60	1353.2	1808.1	2343.7
0.9	654.95	532.58	504.31	567.16	727.56	972.56	1343.2	1799.6	2322.1

Table 6: Simulation results when the proposed estimator $\check{\mathcal{T}}_{21}$ is compared with the estimator $\hat{\hat{Y}}$

ψ \ ϖ_{21}	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
0.1	194.16	162.27	147.18	169.77	218.56	290.84	417.32	531.93	700.09
0.2	226.59	179.69	165.54	192.53	246.92	327.07	458.49	592.66	787.86
0.3	244.65	197.87	182.29	208.92	270.87	359.27	503.59	657.30	870.32
0.4	259.87	210.16	193.93	220.93	284.73	384.16	536.82	700.23	931.24
0.5	266.55	215.90	199.93	226.91	291.38	391.08	548.24	717.8	951.67
0.6	261.83	212.32	196.50	222.57	286.35	384.76	536.17	701.89	930.06
0.7	244.56	200.80	185.50	209.60	269.14	363.34	504.47	663.99	879.04
0.8	224.81	183.61	170.18	191.41	246.68	331.87	458.85	606.02	800.17
0.9	200.96	162.94	151.25	171.04	220.55	296.36	409.10	542.09	714.32

Table 7: Simulation results when the proposed estimator $\check{\mathcal{T}}_{22}$ is compared with the estimator $\hat{\hat{Y}}$

ψ \ ϖ_{22}	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
0.1	471.41	378.45	347.07	417.54	523.23	700.91	985.84	1294.1	1660.1
0.2	528.24	417.18	384.72	451.31	575.64	760.29	1077.0	1413.8	1830.2
0.3	341.14	430.65	402.61	465.7	597.23	788.46	1112.2	1462.5	1926.8
0.4	527.18	420.54	394.10	451.06	576.66	772.73	1081.0	1434.4	1888.2
0.5	485.30	388.39	363.79	412.40	530.19	709.26	990.21	1314.4	1732.8
0.6	425.04	341.89	318.71	361.92	465.0	622.42	863.99	1149.0	1510.5
0.7	355.08	291.67	267.62	306.67	392.04	532.08	733.74	974.43	1290.4
0.8	299.61	244.69	227.33	255.76	329.85	444.27	610.04	811.62	1072.3
0.9	250.56	202.66	188.41	213.37	275.45	370.39	508.81	677.67	893.24

7. Interpretations of Results

7.1. Results from Generalized Empirical Study

a) The optimum values $\mu_{11}^{(0)}, \mu_{12}^{(0)}, \mu_{21}^{(0)}$ and $\mu_{22}^{(0)}$ exist for almost each combination of correlation coefficients. For increasing values of correlation of study and ancillary information, the values $\mu_{11}^{(0)}, \mu_{12}^{(0)}, \mu_{21}^{(0)}$ and $\mu_{22}^{(0)}$ decrease, which in accordance with Sukhatme et al (1984.)

b) As the correlation between study and ancillary information is increased, the percent relative efficiencies increase and the proposed estimators perform better than $\bar{y}_n$ and $\hat{\hat{Y}}$.

c) The proposed estimators provide a lesser fraction of fresh sample drawn afresh as compared to the estimator $\hat{\bar{Y}}$ for almost every considered choice of correlation coefficients.

d) The estimator $\check{\mathcal{T}}_{21}$ performs best in terms of percent relative efficiency and the estimator $\check{\mathcal{T}}_{22}$ performs best in terms of sample drawn afresh at current occasion.

e) As a result, it is also observed that the proposed estimators are working efficiently even for low and moderate correlation values of study and dynamic auxiliary variable on both the occasions.

7.2. Results based on Cost Analysis

a) Theoretically, it is expected that if auxiliary and study variable possess high correlation then this should contribute in reducing the total cost of survey. It is quite evident from the cost analysis that the optimum total cost of the survey decreases for increasing correlation between study and ancillary character.

b) The estimator $\check{\mathcal{T}}_{21}$ and $\check{\mathcal{T}}_{22}$ requires the least total cost for the survey at the current occasion and they both are good in terms of efficiency as well.

7.3 Simulation Results

a) From Table 3to Table 7, it can be seen that the proposed estimators $\check{\mathcal{T}}_{ij}\left(i, j{=}1, 2\right)$ are efficient over $\bar{y}_n$ and $\hat{\bar{Y}}$ for the considered set.

b) Also in simulation study, it is observed that the estimator $\check{\mathcal{T}}_{22}$ is most efficient over the estimators $\bar{y}_n$ and $\hat{\bar{Y}}$ for the considered set.

8. Ratiocination

The entire detailed generalized and simulation studies attest that accompanying dynamic ancillary character with the study character certainly serves the purpose in long lag of two successive waves. The proposed estimators $\check{\mathcal{T}}_{ij}\left(i, j{=}1, 2\right)$ prove to be worthy in terms of precisionas compared to the estimators $\bar{y}_n$ and estimator due to Jessen (1942). The minute observation suggest that the estimators $\check{\mathcal{T}}_{21}$ and $\check{\mathcal{T}}_{22}$ are providing approximately same fraction of sample to be drawn afresh at the current occasion but the total cost of survey is least for the estimator $\check{\mathcal{T}}_{22}$ and $\check{\mathcal{T}}_{21}$ is best in terms of efficiency . Since both the estimators $\check{\mathcal{T}}_{21}$ and $\check{\mathcal{T}}_{22}$ are better than the sample mean estimator and the estimator due to Jessen (1942) but for little amount of precision, the cost of survey cannot be put on stake, therefore $\check{\mathcal{T}}_{22}$ may be regarded as best in terms cost and $\check{\mathcal{T}}_{21}$ may be regarded best in terms of precision. Hence according to the requirement of survey, one is free to choose any of the estimators out of $\check{\mathcal{T}}_{21}$ and $\check{\mathcal{T}}_{22}$. Hence the proposed estimators are recommended to the survey statisticians for their practical use.

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REFERENCES

BANDYOPADHYAY, A., SINGH, G. N., (2014). On the use of two auxiliary variables toimprove the precision of estimate in two-occasion successive sampling, Int. J. Math. Stat., 15, pp.73–88.

ECKLER, A. R., (1955). Rotation Sampling, Ann. Mathe. Stat. 26, pp. 664–685.

SEN, A. R., (1971). Successive sampling with two auxiliary variables. Sankhya, B 33, pp. 371–378.

SEN, A. R., (1972). Successive sampling with p $(p \geq 1)$ auxiliary variables. Ann. Math. Statist. 43, pp. 2031–2034.

SEN, A. R., (1973). Theory and application of sampling on repeated occasions with several auxiliary variables. Biometrics, 29, pp. pp. 381–385.

SINGH, G. N., (2005). On the use of chain-type ratio estimator in successive sampling. Statistics in Transition 7, pp. 21–26.

SINGH, G. N., PRIYANKA, K., (2006). On the use of chain-type ratio to difference estimator in successive sampling. I. J. App. Math. Statis., 5, pp. 41–49.

SINGH, G. N., PRIYANKA, K., (2007). Effect of non-response on current occasion in search of good rotation patterns on successive occasions, Statist. Trans. New Ser. 8, pp. 273–292.

SINGH, G. N., PRIYANKA, K., (2008). Search of good rotation patterns to improve theprecision of estimates at current occasion, Comm. Stat. – Theo. Meth. 37, pp. 337–348.

SINGH, G. N., KARNA, J. P., (2009). Search of efficient rotation patterns in presence of auxiliary information in successive sampling over two occasions. Statis. Tran. N. Ser. 10, pp. 59–73.

SINGH, G. N., PRASAD, S., (2010). Some estimators of population mean in two-occasion rotation patterns. Asso. Adva. Mode. Sim. Tech. Enter., 12, pp. 25–44.

SINGH, G. N., PRIYANKA, K., PRASAD, S., SINGH, S., KIM, J., M., (2013). A class of estimators for estimating the population variance in two occasion rotation patterns,Comm. Statis. App. Meth., 20, pp. 247–257.

PATTERSON, H. D., (1950). Sampling on successive occasions with partial replacement of units, J. Royal Statis. Soci., 12, 241–255.

SINGH, H. P., KUMAR, S., BHOUGAL, S., (2011). Multivariate ratio estimation in presence of non-response in successive sampling. J. Statis. Theo. Prac., 5, pp. 591–611.

GORDON, L., (1983). Successive sampling in finite populations, The Ann. Stat., 11, pp. 702–706.

PRIYANKA, K., MITTAL, R., (2014). Effective rotation patterns for median estimation in successive sampling. Statis. Trans., 15, pp. 197–220.

PRIYANKA, K., MITTAL, R., KIM, J., M., (2015). Multivariate Rotation Design for Population Mean in sampling on Successive Occasions.Comm. Statis. Appli. Meth., 22, pp. 445–462.

PRIYANKA, K., MITTAL, R., (2015a). Estimation of Population Median in Two-OccasionRotation. J. Stat. App. Prob. Lett., 2, pp. 205–219.

PRIYANKA, K., MITTAL, R., (2015 b). A Class of Estimators for Population Median in two Occasion Rotation Sampling. HJMS, 44, pp. 189–202.

SUKHATME, P. V., SUKHATME, B. V., SUKHATME, S., ASOK, C., (1984). Sampling Theory of Surveys with applications. Ames, Iowa State University Press and New Delhi India, Indian Society of Agricultural Statistics.

ARNAB, R., OKAFOR, F. C., (1992). A note on double sampling over two occasions, Paki. J. Stat., 8, pp. 9–18.

NARAIN, R. D., (1953). On the recurrence formula in sampling on successive occasions, J. Indi.Soci. Agri. Stat., 5, pp. 96–99.

JESSEN, R. J., (1942). Statistical investigation of a sample survey for obtaining farm facts, IowaAgri. Exp. Stat.Road Bull., 304, pp. 1–104.

BIRADAR, R. S., SINGH, H. P., (2001). Successive sampling using auxiliary information on both occasions, Cal. Statist. Assoc. Bull., 51, pp. 234–251.

SINGH, R., SINGH, N., (1991). Imputation methods in two-dimensional survey.Recent Advances in Agricultural Statistics Research, Wiley Eastern Ltd, New Delhi,.

SINGH, V. K., SINGH, G. N., SHUKLA, D., (1991). An efficient family of ratio cum difference type estimators in successive sampling over two occasions. J. Sci. Res., 41, pp. 149–159.

FENG, S., ZOU, G., (1997). Sample rotation method with auxiliary variable, Comm. Stat.- Theo. Meth., 26, pp. 1497–1509.

COCHRAN, W., G., (1977). Sampling Techniques, John Wiley & Sons, New Delhi.