STATISTICS IN TRANSITION new series, December 2018
Vol. 19, No. 4, pp. 575–596, DOI 10.21307/stattrans-2018-031

DEVELOPMENT OF CHAIN-TYPE EXPONENTIAL ESTIMATORS FOR POPULATION VARIANCE IN TWO-PHASE SAMPLING DESIGN IN PRESENCE OF RANDOM NON-RESPONSE

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ABSTRACT

In this paper, an investigation has been carried out to deal with a unified approach of estimation procedures of population variance in two-phase sampling design under missing at random non-response mechanism circumstances. Using two auxiliary variables, we have developed different chain-type exponential estimators of finite population variance for two different set-ups and studied their properties under the different assumption of random non-response considered by Tracy and Osahan (1994). The comparisons of the proposed estimators have been made with some contemporary estimators of population variance under the similar realistic conditions. Numerical illustrations are presented to support the theoretical results. Results are analysed and suitable recommendations are put forward to the survey statisticians.

Key words: two-phase sampling, random non-response, variance estimation, study variable, auxiliary information, bias, mean square error

Mathematics Subject Classification: 62D05

1. Introduction

It is well known that in sample surveys the finite population parameter can be estimated more accurately by making use of information on an auxiliary variable x that is correlated with the study variable y. Sometimes, information on auxiliary variable x is not known in advance for all the units of population, for such a situation two-phase sampling is a well-established technique for generating the valid estimates of unknown population parameters of auxiliary variable x in the first phase sample. Ratio, product and regression methods of estimation are good illustrations in this context. Some pioneer works in this direction have been done by several authors, see Chand (1975), Kiregyera (1980), Mukherjee et al. (1987), Singh and Upadhyaya (1995), Pradhan (2005), Singh and Vishwakarma (2007) and Singh and Majhi (2014) among others, in two-phase sampling set-up.

It may be noted that most of the related work of estimation of population variance in sample surveys is based on the assumption of complete response

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from the sample data such as Das and Tripathi (1978), Srivastava and Jhaji (1980), Isaki (1983), Singh (1983), Upadhyay and Singh (1983), Tripathi *et al.* (1988), Biradar and Singh (1994) and Ahmed *et al.* (2003) among others. However, in some practical situations, it is a common experience in sample surveys that the information cannot always be obtained from all the units selected in the sample. For instance, in the first attempt we are not able to collect information from the selected families while some of them may decline to cooperate with the interviewer even if contacted. This results in incomplete data, and this incompleteness is known as non-response and sometimes a huge amount of non-response can completely deviate from desired estimation. Rubin (1976) recommend three particular causes of non-response: missing at random (MAR), observed at random (OAR), and parameter distribution (PD). The missing at random (MAR) response mechanism is helpful in the estimation of population parameters (means, variance, etc.) in economical way even in the presence of non-response in the survey data. Rubin (1976), Tracy and Osahan (1994), Heitzan and Basu (1996), Singh and Joarder (1998), Singh *et al.* (2000) and Singh and Tracy (2001) have suggested the estimators for estimating the finite population parameters (mean, variance, etc.) under the different type random non-response situation. Singh *et al.* (2003), Singh *et al.* (2012) and Bandyopadhyay and Singh (2015) have developed a class of estimators of population variance in two-phase sampling under the situation of random non-response (MAR). Singh *et al.* (2007) studied the properties of a family of estimators for population mean, ratio and product under the above situation of random non-response. Further improvement in the estimation procedure for population variance in the presence of non-response using multi-auxiliary characters in two-phase sampling was suggested by Ahmad *et al.* (2013) under the strategy given by Hansen and Hurwitz (1946).

In the follow-up to the above work and utilizing two auxiliary variables, we have developed some chain-type exponential estimators for estimation of population variance in the presence of random non-response based on missing at random (MAR) response mechanism under two different set-ups of two-phase sampling and studied their properties. The behaviours of the proposed estimators are studied and results are supported with suitable empirical studies, which are followed by suitable recommendation to the survey practitioners.

2. Two-Phase Sampling Structures

Let $U = (U_1, U_2, ..., U_N)$ be the finite population of N units, the character under study is denoted by variable y and the two auxiliary characters are represented by variables x and z respectively with population means $\bar{Y}, \bar{X}$ and $\bar{Z}$. Let y_i, x_i and z_i be the values of y, x and z for the i-th $(i = 1, 2, ..., N)$ unit in the population. To estimate the population variance $S_y^2 = \frac{1}{(N - 1)} \sum_{i=1}^{N} (y_i - \bar{Y})^2$ of study variable y in the presence of auxiliary characters x and z, where the population variance $S_x^2 = \frac{1}{(N - 1)}\sum_{i=1}^{N}\left(x_i - \bar{X}\right)^2$ of x is unknown but the information on z is available for all the units of population, we use the following two-phase sampling scheme. To furnish estimate of population variance S_x^2 of auxiliary variable x, a first phase sample S' of size n is drawn by simple random sampling without replacement (SRSWOR) scheme from the entire population U and observed for the auxiliary characters x and z to estimate S_x^2. Further, a second phase sample S of size m (m <n) is drawn from the first phase sample by the method of simple random sampling without replacement and information on the study variable y and x is gathered.

Case-I: Second phase sample S is drawn as a subsample of the first phase sample $\left(\text{i.e.} S \subset S'\right)$.

Case-II: Second phase sample S is drawn independently of the first phase sample S' $\left(\text{i.e.} S \subset S'\right)$.

3. Non-Response Probability Model

If random non-response situations occur at the second phase sample S of size m and r $\left\{r = 0,1,...., (m - 2)\right\}$ denotes the numbers of sampling units on which information could not be obtained due to random non-response, then the observations of the respective variables on which the random non-response occur can be taken from the remaining responding (m - r) units of the second phase sample. Since we are considering the problem of unbiased estimation of finite population variance S_y^2, therefore it is assumed that r is less than $(m - 1)$, i.e. $0 \le r \le (m - 2)$ and p stands for the probability of non-response among the $(m - 2)$ possible values of non-response, hence r is following discrete distribution; see Singh and Joarder (1998).

$$P(r) = \frac{(m - r)}{mq+2p}\,{}^{m-2}C_r p^r q^{m-2-r}\ ,\ r = 0,1,...., (m - 2) \tag{1}$$

where $q = (1 - p)$.

Here ${}^{m-2}C_r$ denotes the total number of ways of obtaining r non-response out of total possible $(m - 2)$ non-response.

Hence, from now onwards, we use the following notations:

$\bar{Y}$: The population mean of study variable y.

$\bar{X}, \bar{Z}$: The population means of auxiliary variables x and z respectively.

$\bar{y}_m, \bar{x}_m, \bar{x}_n, \bar{z}_m, \bar{z}_n$: The sample means of the respective variables based on the sample sizes shown in the suffices.

$\bar{y}_m^* = \frac{1}{(m-r)}\sum_{i=1}^{m-r} y_i$, $\bar{x}_m^* = \frac{1}{(m-r)}\sum_{i=1}^{m-r} x_i$ and $\bar{z}_m^* = \frac{1}{(m-r)}\sum_{i=1}^{m-r} z_i$: The sample means of the respective variables based on the responding part of the second phase sample S.

$S_z^2 = \frac{1}{(N-1)}\sum_{i=1}^{N}\left(z_i - \bar{Z}\right)^2$: Population variance of the auxiliary variable z.

$s_{y_m}^2 = \frac{1}{(m-1)}\sum_{i=1}^{m}\left(y_i - \bar{y}_m\right)^2$: Sample variance of the study variable y based on sample of size m.

$s_{x_m}^2, s_{x_n}^2$, $s_{z_m}^2$ and $s_{z_n}^2$: Sample variance of the auxiliary characters x and z respectively based on the respective sample sizes shown in their subscripts.

$S_{y_m}^{*2} = \frac{1}{(m-r-1)}\sum_{i=1}^{m-r}\left(y_i - \bar{y}_m^*\right)$: Sample variance of the study variable y based on the responding part of the second phase sample S.

$S_{x_m}^{*2}$ and $S_{z_m}^{*2}$: Sample variance of the auxiliary characters x and z respectively based on the responding part and sample sizes shown in their subscripts.

4. Proposed Strategies

Following the work of Isaki (1983) and utilizing information on an auxiliary variable x with unknown S_x^2, one may propose the ratio type estimator of population variance S_y^2 in two-phase sampling as

$$t_R = s_{y_m}^2 \frac{s_{x_n}^2}{s_{x_m}^2} \qquad (2)$$

Singh and Joarder (1998) have proposed ratio type estimators at the second phase Sample S under random non-response different situation as presented below.

(i) If random non-response occurs only on study variable y at the second phase and the population variance S_x^2 of auxiliary variable x is unknown, then the estimator may be defined as

$$t_1 = s_{y_m}^{*2} \frac{s_{x_n}^2}{s_{x_m}^2} \qquad (3)$$

(ii) If random non-response occurs on both variables y and x and the population variance S_x^2 of auxiliary variable x is unknown, then the estimator may be defined as

$$t_2 = s_{y_m}^{*2} \frac{s_{x_n}^2}{s_{x_m}^{*2}} \tag{4}$$

(iii) In this situation, we consider that random non-response occurs on study variable y as well as auxiliary variables x and z at the second phase sample S and the population variance S_z^2 of auxiliary variable z is unknown, then the estimator may be defined as

$$t_3 = s_{y_m}^{*2} \frac{s_{x_n}^2}{s_{x_m}^{*2}} \frac{s_{z_n}^2}{s_{z_m}^{*2}} \tag{5}$$

(iv) In this situation, we assume that random non-response occurs on study variable y as well as auxiliary variable z at the second phase sample S and the population variance S_z^2 of auxiliary variable z is unknown, then the estimator may be defined as

$$t_4 = s_{y_m}^{*2} \frac{s_{x_n}^2}{s_{x_m}^2} \frac{s_{z_n}^2}{s_{z_m}^{*2}} \tag{6}$$

Following the above suggestions, it is assumed that a complete response situation occurs at the first phase sample S' while non-response situation occurs over all variables y, x and z or in different way in the second phase sample S. We have developed different chain-type exponential estimators of population variance S_y^2 in two-phase sampling design when the population variance S_x^2 of auxiliary variable x is unknown, which may be useful for real life situations such as **(i)**. In the household survey, we considered household size as the auxiliary variable for the estimation of family expenditures. Information may be obtained completely on family size, while there may be random non-response on household expenditure **(ii)**. In the agricultural survey, expenditures of fertilizer or pesticides on crop may be used as the auxiliary variable for estimating the production of crop. There may be random non-response on both the variables. We have presented the following strategies I-IV for handling the above real life situations:

Strategies I: In this situation, we assume that the information on variable y could not be obtained for r units while the complete information on variable x is available at the second phase sample S and the population variance S_z^2 of auxiliary variable z is known. Then, the estimators of finite population variance S_y^2 may be obtained as:

$$T_1 = \frac{s_{y_m}^{*2}}{s_{x_m}^2} s_{x_n}^2 \exp\left\{\frac{S_z^2 - s_{z_n}^2}{S_z^2 + s_{z_n}^2}\right\} \tag{7}$$

$$T_2 = s_{y_m}^{*2} \exp\left\{\frac{s_{x_n}^2 - s_{x_m}^2}{s_{x_n}^2 + s_{x_m}^2}\right\}\left(\frac{S_z^2}{s_{z_n}^2}\right) \quad (8)$$

Strategies II: When random non-response occurs on the study variable y as well as auxiliary variable x at the second phase sample S and the population variance S_z^2 of auxiliary variable z is known. Then, the estimators of finite population variance S_y^2 may be defined as:

$$T_3 = \frac{s_{y_m}^{*2}}{s_{x_m}^{*2}} s_{x_n}^2 \exp\left\{\frac{S_z^2 - s_{z_n}^2}{S_z^2 + s_{z_n}^2}\right\} \quad (9)$$

$$T_4 = s_{y_m}^{*2} \exp\left\{\frac{s_{x_n}^2 - s_{x_m}^{*2}}{s_{x_n}^2 + s_{x_m}^{*2}}\right\}\left(\frac{S_z^2}{s_{z_n}^2}\right) \quad (10)$$

Strategies III: In this situation, it is considered that the random non-response occurs on the study variable y as well as on the auxiliary variables x and z in the second phase sample S and the population variance S_z^2 of the auxiliary variable z is unknown. Then, the estimators of finite population variance S_y^2 may be defined as:

$$T_5 = \frac{s_{y_m}^{*2}}{s_{x_m}^{*2}} s_{x_n}^2 \exp\left\{\frac{s_{z_n}^2 - s_{z_m}^{*2}}{s_{z_n}^2 + s_{z_m}^{*2}}\right\} \quad (11)$$

$$T_6 = s_{y_m}^{*2} \exp\left\{\frac{s_{x_n}^2 - s_{x_m}^{*2}}{s_{x_n}^2 + s_{x_m}^{*2}}\right\}\left(\frac{s_{z_n}^2}{s_{z_m}^{*2}}\right) \quad (12)$$

Strategies IV: In this situation, we assume that the random non-response occurs on the study variable y and the auxiliary variable z with unknown population variance S_z^2 while the complete information on the auxiliary variable x is available. Then, the estimators of finite population variance S_y^2 may be obtained as:

$$T_7 = \frac{s_{y_m}^{*2}}{s_{x_m}^2} s_{x_n}^2 \exp\left\{\frac{s_{z_n}^2 - s_{z_m}^{*2}}{s_{z_n}^2 + s_{z_m}^{*2}}\right\} \quad (13)$$

$$T_8 = s_{y_m}^{*2} \exp\left\{\frac{s_{x_n}^2 - s_{x_m}^2}{s_{x_n}^2 + s_{x_m}^2}\right\}\left(\frac{s_{z_n}^2}{s_{z_m}^{*2}}\right) \qquad (14)$$

5. Properties of Proposed estimators T_i (i = 1, 2,...., 8)

In this section, we derived the bias and mean square errors of the proposed estimators T_i, (i = 1, 2,...., 8) up to the first order of approximation under large sample assumption by using the following transformations:

$$s_{y_m}^{*2} = S_y^2(1+e_0),\ s_{x_m}^{*2} = S_x^2(1+e_1),\ s_{z_m}^{*2} = S_z^2(1+e_2),\ s_{x_m}^2 = S_x^2(1+e_3),$$

$$s_{x_n}^2 = S_x^2(1+e_4),$$

$$s_{z_n}^2 = S_z^2(1+e_5)$$

Such that $|e_i| < 1 \forall$ (i = 1, 2,...., 5)

We have derived the bias and mean square errors of the proposed estimators T_i, (i = 1, 2,...., 8) separately for the cases I and II of the two-phase sampling structure defined in section 2 and present them below.

5.1. Bias and Mean Square Error of proposed estimators under case I

In this section, we have considered that the second phase sample S of size m is drawn as a subsample from the first phase sample S' of size n and we have the following results.

$$E(e_0^2) = f^*C_0^2,\ E(e_1^2) = f^*C_1^2,\ E(e_2^2) = f^*C_2^2,\ E(e_3^2) = f_1C_1^2,$$

$$E(e_4^2) = f_2C_1^2, E(e_5^2) = f_2C_2^2,\ E(e_0e_1) = f^*\rho_{01}C_0C_1,$$

$$E(e_0e_2) = f^*\rho_{02}C_0C_2,\ E(e_0e_3) = f_1\rho_{01}C_0C_1,\ E(e_0e_4) = f_2\rho_{01}C_0C_1,$$

$$E(e_0e_5) = f_2\rho_{02}C_0C_2,\ E(e_1e_2) = f^*\rho_{12}C_1C_2,\ E(e_1e_3) = f_1C_1^2,$$

$$E(e_1e_4) = f_2C_1^2,\ E(e_1e_5) = f_2\rho_{12}C_1C_2,\ E(e_2e_3) = f_1\rho_{12}C_1C_2,$$

$$E(e_2e_4) = f_2\rho_{12}C_1C_2,\ E(e_2e_5) = f_2C_2^2,\ E(e_3e_4) = f_2C_1^2,$$

$$E(e_3e_5) = f_2\rho_{12}C_1C_2,\ E(e_4e_5) = f_2\rho_{12}C_1C_2$$

where $f^{*}=\left(\frac{1}{mq+2p}-\frac{1}{N}\right)$, $f_1=\left(\frac{1}{m}-\frac{1}{N}\right)$, $f_2=\left(\frac{1}{n}-\frac{1}{N}\right)$,

$f_3=\left(\frac{1}{m}-\frac{1}{n}\right)$, $f'=\left(\frac{1}{mq+2p}-\frac{1}{n}\right)$,

$\mu_{rst}=E\left[\left(y_i-\bar{Y}\right)^r\left(x_i-\bar{X}\right)^s\left(z_i-\bar{Z}\right)^t\right]$; $\left(r,s,t\right)\geq 0$ are integers,

$\lambda_{rst}=\mu_{rst}\big/\left(\mu_{200}^{r/2}\mu_{020}^{s/2}\mu_{002}^{t/2}\right)$, $C_0=\sqrt{\left(\lambda_{400}-1\right)}$, $C_1=\sqrt{\left(\lambda_{040}-1\right)}$,

$C_2=\sqrt{\left(\lambda_{004}-1\right)}$, $\rho_{01}=\left(\lambda_{220}-1\right)\big/\sqrt{\left(\lambda_{400}-1\right)\left(\lambda_{040}-1\right)}$,

$\rho_{02}=\left(\lambda_{202}-1\right)\big/\sqrt{\left(\lambda_{400}-1\right)\left(\lambda_{004}-1\right)}$,

$\rho_{12}=\left(\lambda_{022}-1\right)\big/\sqrt{\left(\lambda_{040}-1\right)\left(\lambda_{004}-1\right)}$.

From the above expectations, it is to be noted that:

(a) If p = 0 (there is absence of non-response), the above expected values coincide with the usual results.

(b) ρ_{01} is the correlation coefficient between $\left(y-\bar{Y}\right)^2$ and $\left(x-\bar{X}\right)^2$. Similarly, ρ_{12} is the correlation coefficient between $\left(x-\bar{X}\right)^2$ and $\left(z-\bar{Z}\right)^2$ and ρ_{02} is the correlation coefficient between $\left(y-\bar{Y}\right)^2$ and $\left(z-\bar{Z}\right)^2$; for instance see Upadhyaya and Singh (2006).

Under the above transformations the estimators T_i, (i = 1, 2,...., 8) take the following forms:

$$T_1=S_y^2\left(1+e_0\right)\left(1+e_3\right)^{-1}\left(1+e_4\right)\exp\left\{-\frac{1}{2}e_5\left(1+\frac{e_5}{2}\right)^{-1}\right\} \tag{15}$$

$$T_2=S_y^2\left(1+e_0\right)\left(1+e_5\right)^{-1}\exp\left\{\frac{1}{2}\left(e_4-e_3\right)\left(1+\frac{\left(e_4+e_3\right)}{2}\right)^{-1}\right\} \tag{16}$$

$$T_3=S_y^2\left(1+e_0\right)\left(1+e_1\right)^{-1}\left(1+e_4\right)\exp\left\{-\frac{1}{2}e_5\left(1+\frac{e_5}{2}\right)^{-1}\right\} \tag{17}$$

$$T_4 = S_y^2\left(1+e_0\right)\left(1+e_5\right)^{-1}\exp\left\{\frac{1}{2}\left(e_4 - e_1\right)\left(1+\frac{\left(e_4+e_1\right)}{2}\right)^{-1}\right\} \tag{18}$$

$$T_5 = S_y^2\left(1+e_0\right)\left(1+e_1\right)^{-1}\left(1+e_4\right)\exp\left\{\frac{1}{2}\left(e_5 - e_2\right)\left(1+\frac{\left(e_5+e_2\right)}{2}\right)^{-1}\right\} \tag{19}$$

$$T_6 = S_y^2\left(1+e_0\right)\left(1+e_2\right)^{-1}\left(1+e_5\right)\exp\left\{\frac{1}{2}\left(e_4 - e_1\right)\left(1+\frac{\left(e_4+e_1\right)}{2}\right)^{-1}\right\} \tag{20}$$

$$T_7 = S_y^2\left(1+e_0\right)\left(1+e_3\right)^{-1}\left(1+e_4\right)\exp\left\{\frac{1}{2}\left(e_5 - e_2\right)\left(1+\frac{\left(e_5+e_2\right)}{2}\right)^{-1}\right\} \tag{21}$$

$$T_8 = S_y^2\left(1+e_0\right)\left(1+e_2\right)^{-1}\left(1+e_5\right)\exp\left\{\frac{1}{2}\left(e_4 - e_3\right)\left(1+\frac{\left(e_4+e_3\right)}{2}\right)^{-1}\right\} \tag{22}$$

Now, we expand the right-hand side of equation (15) binomially, up to the first order of approximations, and we have the following expression of the estimator T_1 as:

$$T_1 = S_y^2\left[1+e_0 - e_3+e_4 - \frac{1}{2}e_5+e_3^2+\frac{3}{8}e_5^2 - e_0e_3+e_0e_4 - e_3e_4 - \frac{1}{2}\left(e_0e_5 - e_3e_5+e_4e_5\right)\right] \tag{23}$$

Similarly, we can express the right-hand side of equations (16)-(22) up to the first order of approximations, and we have the expression of the estimators T_i, (i = 2, 3,...., 8).

Taking expectations on both sides of the equation (23) and similarly processing for equations (16)-(22) and then using the expected values of e_i (i = 0, 1,...., 5) , we obtain the bias B(.) and mean square errors M(.) of estimators T_i (i = 1, 2,...., 8) up to the first order of approximations as:

$$B\left(T_1\right) = S_y^2\left[f_3C_1^2+\frac{3}{8}f_2C_2^2 - f_3\rho_{01}C_0C_1 - \frac{1}{2}f_2\rho_{02}C_0C_2\right] \tag{24}$$

$$B\left(T_2\right) = S_y^2\left[f_2C_2^2+\frac{3}{8}f_3C_1^2 - f_2\rho_{02}C_0C_2 - \frac{1}{2}f_3\rho_{01}C_0C_1\right] \tag{25}$$

$$B\left(T_3\right) = S_y^2\left[f'C_1^2+\frac{3}{8}f_2C_2^2 - f'\rho_{01}C_0C_1 - \frac{1}{2}f_2\rho_{02}C_0C_2\right] \tag{26}$$

$$B(T_4) = S_y^2\left[f_2C_2^2+\frac{3}{8}f'C_2^2 - f_2\rho_{02}C_0C_2 - \frac{1}{2}f'\rho_{01}C_0C_1\right] \tag{27}$$

$$B(T_5) = S_y^2\left[f'C_1^2+\frac{3}{8}f'C_2^2 - f'\rho_{01}C_0C_1 - \frac{1}{2}f'C_2\left(\rho_{02}C_0 - \rho_{12}C_1\right)\right] \tag{28}$$

$$B(T_6) = S_y^2\left[f'C_2^2+\frac{3}{8}f'C_1^2 - f'\rho_{02}C_0C_2 - \frac{1}{2}f'C_1\left(\rho_{01}C_0 - \rho_{12}C_2\right)\right] \tag{29}$$

$$B(T_7) = S_y^2\left[f_3C_1^2+\frac{3}{8}f'C_2^2 - f_3\rho_{01}C_0C_1 - \frac{1}{2}f'\rho_{02}C_0C_2+\frac{1}{2}f_3\rho_{12}C_1C_2\right] \tag{30}$$

$$B(T_8) = S_y^2\left[f'C_2^2+\frac{3}{8}f_3C_1^2 - f'\rho_{02}C_0C_2 - \frac{1}{2}f_3C_1\left(\rho_{01}C_0 - \rho_{12}C_2\right)\right] \tag{31}$$

$$M(T_1) = S_y^4\left[f^*C_0^2+f_3C_1^2+\frac{1}{4}f_2C_2^2 - 2f_3\rho_{01}C_0C_1 - f_2\rho_{02}C_0C_2\right] \tag{32}$$

$$M(T_2) = S_y^4\left[f^*C_0^2+f_2C_2^2+\frac{1}{4}f_3C_1^2 - f_3\rho_{01}C_0C_1 - 2f_2\rho_{02}C_0C_2\right] \tag{33}$$

$$M(T_3) = S_y^4\left[f^*C_0^2+f'C_1^2+\frac{1}{4}f_2C_2^2 - 2f'\rho_{01}C_0C_1 - f_2\rho_{02}C_0C_2\right] \tag{34}$$

$$M(T_4) = S_y^4\left[f^*C_0^2+f_2C_2^2+\frac{1}{4}f'C_1^2 - f'\rho_{01}C_0C_1 - 2f_2\rho_{02}C_0C_2\right] \tag{35}$$

$$M(T_5) = S_y^4\left[f^*C_0^2+f'C_1^2+\frac{1}{4}f'C_2^2 - f'\rho_{02}C_0C_2+f'\rho_{12}C_1C_2 - 2f'\rho_{01}C_0C_1\right] \tag{36}$$

$$M(T_6) = S_y^4\left[f^*C_0^2+f'C_2^2+\frac{1}{4}f'C_1^2 - f'\rho_{01}C_0C_1+f'\rho_{12}C_1C_2 - 2f'\rho_{02}C_0C_2\right] \tag{37}$$

$$M(T_7) = S_y^4\left[f^*C_0^2+f_3C_1^2+\frac{1}{4}f'C_2^2 - f'\rho_{02}C_0C_2+f_3\rho_{12}C_1C_2 - 2f_3\rho_{01}C_0C_1\right] \tag{38}$$

and

$$M(T_8) = S_y^4\left[f^*C_0^2+f'C_2^2+\frac{1}{4}f_3C_1^2 - f_3\rho_{01}C_0C_1+f_3\rho_{12}C_1C_2 - 2f'\rho_{02}C_0C_2\right] \tag{39}$$

5.2. Bias and Mean Square Error of proposed estimators under case II

If the second phase sample S is drawn independently of the first phase sample S', then we have the following results.

$$E\left(e_0^2\right)=f^*C_0^2,\ E\left(e_1^2\right)=f^*C_1^2,\ E\left(e_2^2\right)=f^*C_2^2,\ E\left(e_3^2\right)=f_1C_1^2,\ E\left(e_4^2\right)=f_2C_1^2$$

$$,\ E\left(e_5^2\right)=f_2C_2^2,\ E\left(e_0e_1\right)=f^*\rho_{01}C_0C_1,\ E\left(e_0e_2\right)=f^*\rho_{02}C_0C_2,$$

$$E\left(e_0e_3\right)=f_1\rho_{01}C_0C_1,\ E\left(e_1e_2\right)=f^*\rho_{12}C_1C_2,\ E\left(e_1e_3\right)=f_1C_1^2,$$

$$E\left(e_2e_3\right)=f_1\rho_{12}C_1C_2,\ E\left(e_4e_5\right)=f_2\rho_{12}C_1C_2$$

$$E\left(e_0e_4\right)=E\left(e_0e_5\right)=E\left(e_1e_4\right)=E\left(e_1e_5\right)=E\left(e_2e_4\right)=E\left(e_2e_5\right)=E\left(e_3e_4\right)=E\left(e_3e_5\right)=0$$

Proceeding as section 5.1 and using the expected value as section 5.2, we have derived the expressions for bias B(.) and mean square errors M(.) of the proposed estimators T_i (i = 1, 2,...., 8) to the first order of approximations as:

$$B\left(T_1\right)=S_y^2\left[f_1C_1^2+\frac{3}{8}f_2C_2^2-f_1\rho_{01}C_0C_1-\frac{1}{2}f_2\rho_{02}C_0C_2\right] \quad (40)$$

$$B\left(T_2\right)=S_y^2\left[f_2C_2^2+\frac{1}{8}C_1^2\left(3f_1-f_2\right)-\frac{1}{2}\left(f_1\rho_{01}C_0C_1+f_2\rho_{12}C_1C_2\right)\right] \quad (41)$$

$$B\left(T_3\right)=S_y^2\left[f^*C_1^2+\frac{3}{8}f_2C_2^2-f^*\rho_{01}C_0C_1-\frac{1}{2}f_2\rho_{12}C_1C_2\right] \quad (42)$$

$$B\left(T_4\right)=S_y^2\left[f_2C_2^2+\frac{1}{8}C_1^2\left(3f^*-f_2\right)-\frac{1}{2}C_1\left(f^*\rho_{01}C_0+f_2\rho_{12}C_2\right)\right] \quad (43)$$

$$B\left(T_5\right)=S_y^2\left[f^*C_1^2+\frac{1}{8}C_2^2\left(3f^*-f_2\right)-f^*\rho_{01}C_0C_1+\frac{1}{2}\left\{\left(f_2+f^*\right)\rho_{12}C_1C_2-f^*\rho_{02}C_0C_2\right\}\right] \quad (44)$$

$$B\left(T_6\right)=S_y^2\left[f^*C_2^2+\frac{1}{8}C_1^2\left(3f^*-f_2\right)-f^*\rho_{02}C_0C_2+\frac{1}{2}\left\{\left(f_2+f^*\right)\rho_{12}C_1C_2-f^*\rho_{01}C_0C_1\right\}\right] \quad (45)$$

$$B\left(T_7\right)=S_y^2\left[f_1C_1^2+\frac{1}{8}C_2^2\left(3f^*-f_2\right)-f_1\rho_{01}C_0C_1+\frac{1}{2}\left\{\left(f_1+f_2\right)\rho_{12}C_1C_2-f^*\rho_{02}C_0C_2\right\}\right] \quad (46)$$

$$B(T_8)=S_y^2\left[f^*C_2^2+\frac{1}{8}C_1^2(3f_1-f_2)-f^*\rho_{02}C_0C_2+\frac{1}{2}\left\{(f_1+f_2)\rho_{12}C_1C_2-f_1\rho_{01}C_0C_1\right\}\right] \tag{47}$$

$$M(T_1)=S_y^4\left[f^*C_0^2+(f_1+f_2)C_1^2+\frac{1}{4}f_2C_2^2-2f_1\rho_{01}C_0C_1-f_2\rho_{12}C_1C_2\right] \tag{48}$$

$$M(T_2)=S_y^4\left[f^*C_0^2+f_2C_2^2+\frac{1}{4}(f_1+f_2)C_1^2-f_1\rho_{01}C_0C_1-f_2\rho_{12}C_1C_2\right] \tag{49}$$

$$M(T_3)=S_y^4\left[f^*C_0^2+(f^*+f_2)C_1^2+\frac{1}{4}f_2C_2^2-2f^*\rho_{01}C_0C_1-f_2\rho_{12}C_1C_2\right] \tag{50}$$

$$M(T_4)=S_y^4\left[f^*C_0^2+f_2C_2^2+\frac{1}{4}(f^*+f_2)C_1^2-f^*\rho_{01}C_0C_1-f_2\rho_{12}C_1C_2\right] \tag{51}$$

$$M(T_5)=S_y^4\left[f^*C_0^2+(f^*+f_2)\left(C_1^2+\frac{1}{4}C_2^2+\rho_{12}C_1C_2\right)-f^*\left(\rho_{02}C_0C_2+2\rho_{01}C_0C_1\right)\right] \tag{52}$$

$$M(T_6)=S_y^4\left[f^*C_0^2+(f^*+f_2)\left(C_2^2+\frac{1}{4}C_1^2+\rho_{12}C_1C_2\right)-f^*\left(\rho_{01}C_0C_1+2\rho_{02}C_0C_2\right)\right] \tag{53}$$

$$M(T_7)=S_y^4\left[f^*C_0^2+(f_1+f_2)\left(C_1^2+\rho_{12}C_1C_2\right)+\frac{1}{4}(f^*+f_2)C_2^2-f^*\rho_{02}C_0C_2-2f_1\rho_{01}C_0C_1\right] \tag{54}$$

and

$$M(T_8)=S_y^4\left[f^*C_0^2+(f^*+f_2)C_2^2+(f_1+f_2)\left(\frac{1}{4}C_1^2+\rho_{12}C_1C_2\right)-f_1\rho_{01}C_0C_1-2f^*\rho_{02}C_0C_2\right] \tag{55}$$

6. Efficiency Comparisons of the Proposed Estimators T_i (i = 1, 2,...., 8)

In this section, we validate the performance of the proposed estimators T_i (i = 1, 2,..., 8) with to respect to the estimators such as population variance $s_{y_m}^{*2}$

(sample variance estimator in presence of random non-response) and t_i (i = 1, 2, ..., 4). The variance/mean square errors of the estimators $s_{y_m}^{*2}$ and t_i (i = 1, 2, ..., 4) up to the first order of approximation under case I and case II are respectively given by:

Case I:

$$V\left(s_{y_m}^{*2}\right) = f^{*}C_0^2S_y^4 \tag{56}$$

$$M(t_1) = S_y^4\left[f^{*}C_0^2+f_3C_1^2 -2f_3\rho_{01}C_0C_1\right] \tag{57}$$

$$M(t_2) = S_y^4\left[f^{*}C_0^2+f'\left(C_1^2 -2\rho_{01}C_0C_1\right)\right] \tag{58}$$

$$M(t_3) = S_y^4\left[f^{*}C_0^2+f'\left(C_1^2+C_2^2\right) -2f'\left(\rho_{01}C_0C_1+\rho_{02}C_0C_2 -\rho_{12}C_1C_2\right)\right] \tag{59}$$

and

$$M(t_4) = S_y^4\left[f^{*}C_0^2+f_3\left(C_1^2 -2\rho_{01}C_0C_1+2\rho_{12}C_1C_2\right)+f'\left(C_2^2 -2\rho_{02}C_0C_2\right)\right] \tag{60}$$

Case II:

$$V\left(s_{y_m}^{*2}\right) = f^{*}C_0^2S_y^4 \tag{60}$$

$$M(t_1) = S_y^4\left[f^{*}C_0^2+\left(f_1+f_2\right)C_1^2 -2f_1\rho_{01}C_0C_1\right] \tag{61}$$

$$M(t_2) = S_y^4\left[f^{*}C_0^2+\left(f^{*}+f_2\right)C_1^2 -2f^{*}\rho_{01}C_0C_1\right] \tag{62}$$

$$M(t_3) = S_y^4\left[f^{*}C_0^2+\left(f^{*}+f_2\right)\left(C_1^2+C_2^2+2\rho_{12}C_1C_2\right) -2f^{*}\left(\rho_{01}C_0C_1+\rho_{02}C_0C_2\right)\right] \tag{63}$$

and

$$M(t_4) = S_y^4\left[f^{*}C_0^2+\left(f_1+f_2\right)\left(C_1^2+2\rho_{12}C_1C_2\right)+\left(f^{*}+f_2\right)C_2^2 -2f_1\rho_{01}C_0C_1 -2f^{*}\rho_{02}C_0C_2\right] \tag{64}$$

The performances of our proposed estimators T_i (i = 1, 2,...., 8) are compared with the other estimators considered in this paper and their dominance is examined below through empirical studies carried over three different populations.

7. Numerical Illustration

We have computed the percent relative efficiencies of the proposed estimators T_i (i = 1, 2,...., 8) with respect to $s_{y_m}^{*2}$ and t_i (i = 1, 2, ..., 4) based on three natural populations. The source of the populations, the nature of the variables y, x, z and the values of the various parameters are given as follows.

Population I- Source: Sukhatme and Sukhatme [1970] (page-185)

y: Area under wheat in 1937.

x: Area under wheat in 1936.

z: Total cultivated area in 1931.

N = 34, $C_0 = 1.5959$, $C_1 = 1.5105$, $C_2 = 1.3200$,
$\rho_{01} = 0.6251$, $\rho_{02} = 0.8007$, $\rho_{12} = 0.5342$

Population II- Source: Murthy[1967] (page-399)

y: Area under wheat in 1964.

x: Area under wheat in 1963.

z: Total cultivated area in 1961.

N 33= 34, $C_0 = 1.6510$, $C_1 = 1.3828$, $C_2 = 1.3447$,
$\rho_{01} = 0.9218$, $\rho_{02} = 0.8914$, $\rho_{12} = 0.9346$

Population III- Source: Satici and Kadilar (2011)

This data set is about 923 district of Turkey.

y: Number of successful students.

x: Numbers of teachers.

z: Private teaching institutions.

N = 261, $C_0 = 1.86537$, $C_1 = 1.75941$, $C_2 = 2.02126$,
$\rho_{01} = 0.970$, $\rho_{02} = 0.935$, $\rho_{12} = 0.928$

Table 1. Percent relative efficiency of the estimators T_1 and T_2 with respect to other estimators when non-response situation occur only on study variable y at the second phase sample.

Population I								
Estimators	Case I				Case II			
	T_1		T_2		T_1		T_2	
	$s_{y_m}^{*2}$	t_1	$s_{y_m}^{*2}$	t_1	$s_{y_m}^{*2}$	t_1	$s_{y_m}^{*2}$	t_1
P=0.02	143.93	108.55	164.43	124.02	125.32	103.74	142.77	118.19
P=0.04	142.41	108.26	161.91	123.08	124.55	103.63	141.30	117.57
P=0.06	140.95	107.97	159.49	122.18	123.81	103.52	139.89	116.97
P=0.08	139.54	107.70	157.19	121.32	123.09	103.41	138.53	116.39
P=0.10	138.18	107.43	154.98	120.49	122.38	103.31	137.21	115.83
Population II								
Estimators	Case I				Case II			
	T_1		T_2		T_1		T_2	
	$s_{y_m}^{*2}$	t_1	$s_{y_m}^{*2}$	t_1	$s_{y_m}^{*2}$	t_1	$s_{y_m}^{*2}$	t_1
P=0.02	472.17	132.00	253.54	*	486.82	127.79	226.25	*
P=0.04	433.14	128.65	244.45	*	445.12	124.79	219.54	*
P=0.06	400.53	125.84	236.12	*	410.51	122.30	213.30	*
P=0.08	372.89	123.47	228.45	*	381.30	120.20	207.50	*
P=0.10	349.15	121.42	221.36	*	356.33	118.41	202.09	*
Population III								
Estimators	Case I				Case II			
	T_1		T_2		T_1		T_2	
	$s_{y_m}^{*2}$	t_1	$s_{y_m}^{*2}$	t_1	$s_{y_m}^{*2}$	t_1	$s_{y_m}^{*2}$	t_1
P=0.02	752.54	230.81	352.72	108.18	733.68	216.06	232.38	*
P=0.04	660.37	212.33	334.63	107.59	646.11	200.02	225.98	*
P=0.06	588.63	197.95	318.38	107.07	577.51	187.46	219.96	*
P=0.08	531.19	186.44	303.71	106.59	522.32	177.35	214.27	*
P=0.10	484.18	177.01	290.38	106.16	476.96	169.04	208.89	*

* Indicate, proposed estimator is not preferable over existing estimator.

Table 2. Percent relative efficiency of the estimators T_3 and T_4 with respect to other estimators when non-response situation occur on study variable y and auxiliary variable x at the second phase sample.

Population I

Estimators	Case I				Case II			
	T_3		T_4		T_3		T_4	
	$s_{y_m}^{*2}$	t_2	$s_{y_m}^{*2}$	t_2	$s_{y_m}^{*2}$	t_2	$s_{y_m}^{*2}$	t_2
P=0.02	145.37	108.64	166.86	124.69	126.41	103.78	144.59	118.70
P=0.04	145.24	108.42	166.63	124.39	126.72	103.69	144.89	118.57
P=0.06	145.12	108.21	166.41	124.09	127.02	103.61	145.19	118.43
P=0.08	145.00	108.00	166.20	123.79	127.31	103.53	145.48	118.30
P=0.10	144.88	107.79	165.99	123.50	127.61	103.45	145.77	118.17

Population II

Estimators	Case I				Case II			
	T_3		T_4		T_3		T_4	
	$s_{y_m}^{*2}$	t_2	$s_{y_m}^{*2}$	t_2	$s_{y_m}^{*2}$	t_2	$s_{y_m}^{*2}$	t_2
P=0.02	522.02	135.38	263.10	*	539.98	130.82	233.82	*
P=0.04	524.28	134.68	262.71	*	541.95	130.18	234.15	*
P=0.06	526.53	133.97	262.32	*	543.91	129.55	234.46	*
P=0.08	528.77	133.28	261.95	*	545.84	128.93	234.78	*
P=0.10	530.99	132.58	261.58	*	547.77	128.30	235.09	*

Population III

Estimators	Case I				Case II			
	T_3		T_4		T_3		T_4	
	$s_{y_m}^{*2}$	t_2	$s_{y_m}^{*2}$	t_2	$s_{y_m}^{*2}$	t_2	$s_{y_m}^{*2}$	t_2
P=0.02	884.16	253.69	371.83	106.69	858.24	235.77	240.53	*
P=0.04	893.16	251.93	370.69	104.56	867.27	234.26	241.87	*
P=0.06	902.30	250.15	369.57	102.45	876.44	232.73	243.23	*
P=0.08	911.59	248.34	368.45	100.37	885.76	231.17	244.59	*
P=0.10	921.02	246.50	367.35	98.31	895.24	229.59	245.97	*

Table 3. Percent relative efficiency of the estimators T_5 and T_6 with respect to other estimators when non-response situation occur on study variable y as well as auxiliary variable x and z at the second phase sample

Population I

Estimators	Case I				Case II			
	T_5		T_6		T_5		T_6	
	$s_{y_m}^{*2}$	t_3	$s_{y_m}^{*2}$	t_3	$s_{y_m}^{*2}$	t_3	$s_{y_m}^{*2}$	t_3
P=0.02	146.38	134.61	207.69	190.99	122.06	146.59	175.25	210.47
P=0.04	146.61	134.78	208.44	191.62	122.71	146.51	176.45	210.66
P=0.06	146.83	134.95	209.18	192.25	123.36	146.42	177.65	210.86
P=0.08	147.05	135.11	209.91	192.87	124.01	146.33	178.85	211.05
P=0.10	147.27	135.27	210.64	193.49	124.65	146.25	180.05	211.24

Population II

Estimators	Case I				Case II			
	T_5		T_6		T_5		T_6	
	$s_{y_m}^{*2}$	t_3	$s_{y_m}^{*2}$	t_3	$s_{y_m}^{*2}$	t_3	$s_{y_m}^{*2}$	t_3
P=0.02	305.47	209.83	291.80	200.44	239.88	231.07	232.09	223.57
P=0.04	307.57	210.96	293.68	201.43	242.44	231.67	234.45	224.03
P=0.06	309.68	212.08	295.55	202.41	245.02	232.27	236.81	224.48
P=0.08	311.78	213.20	297.42	203.39	247.62	232.87	239.19	224.94
P=0.10	313.87	214.32	299.28	204.36	250.23	233.47	241.58	225.40

Population III

Estimators	Case I				Case II			
	T_5		T_6		T_5		T_6	
	$s_{y_m}^{*2}$	t_3	$s_{y_m}^{*2}$	t_3	$s_{y_m}^{*2}$	t_3	$s_{y_m}^{*2}$	t_3
P=0.02	217.16	234.38	182.54	197.01	124.44	256.51	103.06	212.44
P=0.04	218.91	236.38	183.57	198.23	126.17	257.50	104.37	212.99
P=0.06	220.67	238.41	184.61	199.45	127.94	258.50	105.70	213.56
P=0.08	222.46	240.46	185.66	200.68	129.75	259.53	107.05	214.13
P=0.10	224.27	242.53	186.71	201.91	131.61	260.58	108.44	214.72

Table 4. Percent relative efficiency of the estimators T_7 and T_8 with respect to other estimators when non-response situation occur on study variable y and auxiliary variable z at the second phase sample

Population I

Estimators	Case I				Case II			
	T_7		T_8		T_7		T_8	
	$s_{y_m}^{*2}$	t_4	$s_{y_m}^{*2}$	t_4	$s_{y_m}^{*2}$	t_4	$s_{y_m}^{*2}$	t_4
P=0.02	147.06	133.29	208.21	188.73	122.53	145.54	175.62	208.60
P=0.04	147.96	132.15	209.49	187.11	123.66	144.40	177.20	206.93
P=0.06	148.86	131.01	210.76	185.50	124.79	143.26	178.79	205.25
P=0.08	149.75	129.88	212.02	183.89	125.92	142.11	180.38	203.56
P=0.10	150.65	128.75	213.29	182.28	127.06	140.96	181.97	201.87

Population II

Estimators	Case I				Case II			
	T_7		T_8		T_7		T_8	
	$s_{y_m}^{*2}$	t_4	$s_{y_m}^{*2}$	t_4	$s_{y_m}^{*2}$	t_4	$s_{y_m}^{*2}$	t_4
P=0.02	300.94	203.60	292.64	197.98	237.08	225.91	232.62	221.67
P=0.04	298.60	198.65	295.37	196.50	236.83	221.43	235.52	220.20
P=0.06	296.33	193.85	298.11	195.01	236.59	217.02	238.45	218.72
P=0.08	294.13	189.20	300.85	193.52	236.35	212.69	241.40	217.23
P=0.10	292.00	184.69	303.61	192.03	236.12	208.43	244.38	215.72

Population III

Estimators	Case I				Case II			
	T_7		T_8		T_7		T_8	
	$s_{y_m}^{*2}$	t_4	$s_{y_m}^{*2}$	t_4	$s_{y_m}^{*2}$	t_4	$s_{y_m}^{*2}$	t_4
P=0.02	217.24	230.09	184.35	195.26	124.47	254.06	103.64	211.54
P=0.04	219.07	227.76	187.27	194.69	126.23	252.54	105.55	211.17
P=0.06	220.93	225.40	190.26	194.11	128.03	250.98	107.52	210.79
P=0.08	222.80	223.02	193.33	193.52	129.87	249.38	109.56	210.39
P=0.10	224.70	220.60	196.49	192.91	131.75	247.75	111.67	209.98

The empirical studies are carried out for different choices of non-response rate p, the performances of the proposed estimators T_i (i = 1, 2,...., 8) have been shown in terms of the percent relative efficiencies with respect to other estimators.

$$PRE = \frac{M(\delta)}{M(T_i)} \times 100 \text{, (i = 1, 2 ,..., 8)}$$

8. Interpretations of Empirical Results

The following interpretations can be read out from the present study.

From Tables 1-4, it is visible that almost all the values of percent relative efficiencies are exceeding 100 for all the parametric combinations, which indicate that the proposed estimators are uniformly dominating over the existing estimators as considered in this work.

(a) From Tables 1 and 3, it may be seen that the values of percent relative efficiencies decrease and increase respectively for both the cases as the values of non-response rate p increase.

(b) Further, when the random non-response rate p increases we observe the zig-zag trend in Tables 2 and 4.

9. Conclusions

In this paper, we have studied different chain-type exponential estimators for improving estimation of the population variance under the situation of random non-response. Following the analyses of effective estimation procedures, it has been found that the results are highly desirable, which indicate the proposition of proposed estimators and subsequent estimation procedures. Hence, looking on the nice behaviour, the proposed estimation procedures may be recommended to the survey statisticians for their practical application whenever they intend to deal with the sensitive or stigmatizing attributes such as drinking alcohol, gambling habit, drug addiction, tax evasion, history of induced abortions, etc.

Acknowledgements

The authors are thankful to the reviewers for their valuable suggestions, which helped in improving the quality of this paper. The authors are also grateful to the IIT(ISM), Dhanbad for providing the financial assistance and necessary infrastructure to carry out the present research work.

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