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Some linear regression type ratio exponential estimators for estimating the population mean based on quartile deviation and deciles

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ABSTRACT

This paper deals some linear regression type ratio exponential estimators for estimating the population mean using the known values of quartile deviation and deciles of an auxiliary variable in survey sampling. The expressions of the bias and the mean square error of the suggested estimators have been derived. It was compared with the usual mean, usual ratio (Cochran (1977)), Kadilar and Cingi (2004, 2006) and Subzar *et al.* (2017) estimators. After comparison, the condition which makes the suggested estimators more efficient than others is found. To verify the theoretical results, numerical results are performed on two natural population data sets.

Key words: Bias, Mean square error (MSE), Auxiliary variable, Relative Efficiency (%).

1. Introduction

Cochran (1977) considered a classical ratio type estimator for the estimation of finite population mean by using auxiliary information when the coefficient of correlation between auxiliary variable X and the estimated variable Y is positive. Sisodia and Dwivedi (1981) utilized the coefficient of variation of the auxiliary variable in survey sampling. Upadhyaya and Singh (1999) modified ratio type estimators using the coefficient of variation and the coefficient of kurtosis of the auxiliary variable. Yan and Tian (2010), Subramani and Kumarapandian (2012 (a), 2012 (b), 2012 (c), 2012 (d)), Swain (2014) and Abid *et al.* (2016 (a), 2016 (b), 2016 (c)) etc, considered a large number of modified ratio estimators using the known values of population parameters of auxiliary variable in survey sampling. Recently Subzar *et al.* (2017) considered new ratio type estimators in simple random sampling by using the conventional location parameters.

The paper is structured as follows. In Section 2, the existing and studied so far linear regression type ratio estimators are presented. In Section 3, newly proposed classes of estimators are formally presented. The properties of the suggested estimators are discussed in Section 4. The theoretical comparisons between the suggested estimators and the other existing estimators are considered in Section 5. A numerical demonstration is conducted in Section 6 to support and verify the theoretical results and some concluding remarks are given in Section 7.

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2. Brief Description of Some Existing Estimators

Let y and x be denoted by the positively correlated study variable and auxiliary variable respectively. A simple random sample (without replacement) s_n of n units is drawn from a finite population U = (U_1, U_2, ..., U_N) of N units to estimate population mean $\bar{Y}$, which uses the known values of population parameters of auxiliary variable such as quartile deviation and deciles. The following notations have been approached in this work:
$\bar{Y}, \bar{X}$: The population means of the variables y and x respectively.
$S_x^2 = (N-1)^{-1}\sum_{i=1}^{N}(x_i-\bar{X})^2$: The population variance of the variable x.
S_y^2 : The population variance of the variable y.
$S_{yx} = \frac{1}{N-1}\sum_{i=1}^{N}(y_i-\bar{Y})(x_i-\bar{X})$: The population covariance between the variables y and x.
C_y and C_x : The coefficients of variation of the variables y and x respectively.
$\beta_1(x)$: The population coefficient of skewness of the variable x.
$\beta_2(x)$: The population coefficient of kurtosis of the variable x.
ρ : The Pearson correlation coefficient between the variables y and x.
$D_i, i = 1,2,...,10$: Population Deciles.
$QD = \frac{Q_3-Q_1}{2}$: Population quartile deviation.
In this section, several ratio type estimators have been considered for estimating the population mean in survey sampling:

2.1. Usual Mean Estimator

The estimator of sample mean $\bar{y}$ is derived as $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$, which is known as the usual unbiased estimator $\bar{y}$ of population mean $\bar{Y}$. The variance of the sample mean $\bar{y}$, is given by $Var(\bar{y}) = \left(\frac{1}{n}-\frac{1}{N}\right)\bar{Y}^2 C_y^2$.

2.2. Usual Ratio (Cochran (1977)) Estimator

Cochran (1977) considered the ratio estimator of population mean $\bar{Y}$ as $\bar{y}_{Ratio} = \bar{y}\frac{\bar{X}}{\bar{x}}, (\bar{x} \neq 0)$. The MSE of estimator $\bar{y}_{Ratio}$, is given by
$MSE(\bar{y}_{Ratio}) = \left(\frac{1}{n}-\frac{1}{N}\right)\bar{Y}^2\left(C_y^2 - 2\rho C_y C_x + C_x^2\right)$.

2.3. Kadilar and Cingi (2004) Estimators

Kadilar and Cingi (2004) considered the following ratio estimators for the population mean of the study variable $\bar{Y}$ by using auxiliary information in survey sampling:

$T_{KC(1)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}}\bar{X}$, $T_{KC(2)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}+C_x}(\bar{X}+C_x)$,

$T_{KC(3)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}+\beta_2(x)}(\bar{X}+\beta_2(x))$, $T_{KC(4)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}\beta_2(x)+C_x}(\bar{X}\beta_2(x)+C_x)$,

$T_{KC(5)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}C_x+\beta_2(x)}(\bar{X}C_x+\beta_2(x))$,

where $\hat{\beta} = \frac{s_{yx}}{s_x^2}$, $s_{yx} = \frac{1}{n-1}\sum_{i=1}^{n}(y_i-\bar{y})(x_i-\bar{x})$, $s_x^2 = (n-1)^{-1}\sum_{i=1}^{n}(x_i-\bar{x})^2$.
The MSEs of the estimators $T_{KC(1)}$, $T_{KC(2)}$, $T_{KC(3)}$, $T_{KC(4)}$ and $T_{KC(5)}$, are given by
$MSE(T_{KC(i)}) = \left(\frac{1}{n}-\frac{1}{N}\right)\left(KC_i^2 C_x^2 + \left(1-\rho^2\right)C_y^2\right)\bar{Y}^2$, where $(i = 1,2,...,5)$,
$KC_1 = 1$, $KC_2 = \frac{\bar{X}}{\bar{X}+C_x}$, $KC_3 = \frac{\bar{X}}{\bar{X}+\beta_2(x)}$, $KC_4 = \frac{\bar{X}\beta_2(x)}{\bar{X}\beta_2(x)+C_x}$, $KC_5 = \frac{C_x\bar{X}}{C_x\bar{X}+\beta_2(x)}$.

2.4. Kadilar and Cingi (2006) Estimators

Kadilar and Cingi (2006) considered the following ratio estimators for the population mean of the study variable $\bar{Y}$ by using the coefficient of correlation in survey sampling:

$T_{KC(6)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}+\rho}(\bar{X}+\rho)$, $T_{KC(7)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}C_x+\rho}(\bar{X}C_x+\rho)$,

$T_{KC(8)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}\rho+C_x}(\bar{X}\rho+C_x)$, $T_{KC(9)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}\beta_2(x)+\rho}(\bar{X}\beta_2(x)+\rho)$,

$T_{KC(10)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}\rho+\beta_2(x)}(\bar{X}\rho+\beta_2(x))$.

The MSEs of the estimators $T_{KC(6)}, T_{KC(7)}, T_{KC(8)}, T_{KC(9)}$ and $T_{KC(10)}$, are given by $MSE(T_{KC(i)}) = \left(\frac{1}{n}-\frac{1}{N}\right)\left(KC_i^2C_x^2+\left(1-\rho^2\right)C_y^2\right)\bar{Y}^2$, where $(i=6,7,...,10)$,
$KC_6 = \frac{\bar{X}}{\bar{X}+\rho}$, $KC_7 = \frac{\bar{X}C_x}{\bar{X}C_x+\rho}$, $KC_8 = \frac{\bar{X}\rho}{\bar{X}\rho+C_x}$, $KC_9 = \frac{\bar{X}\beta_2(x)}{\bar{X}\beta_2(x)+\rho}$, $KC_{10} = \frac{\rho\bar{X}}{\bar{X}\rho+\beta_2(x)}$.

2.5. Subzar *et al.* (2017) Estimators

Subzar *et al.* (2017) proposed a new ratio estimators for estimation of the population mean $\bar{Y}$ as

$T_{Smrs(1)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}QD+D_1}(\bar{X}QD+D_1)$, $T_{Smrs(2)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}QD+D_2}(\bar{X}QD+D_2)$,

$T_{Smrs(3)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}QD+D_3}(\bar{X}QD+D_3)$, $T_{Smrs(4)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}QD+D_4}(\bar{X}QD+D_4)$,

$T_{Smrs(5)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}QD+D_5}(\bar{X}QD+D_5)$, $T_{Smrs(6)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}QD+D_6}(\bar{X}QD+D_6)$,

$T_{Smrs(7)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}QD+D_7}(\bar{X}QD+D_7)$, $T_{Smrs(8)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}QD+D_8}(\bar{X}QD+D_8)$,

$T_{Smrs(9)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}QD+D_9}(\bar{X}QD+D_9)$, $T_{Smrs(10)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}QD+D_{10}}(\bar{X}QD+D_{10})$,

$T_{Smrs(11)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}D_1+QD}(\bar{X}D_1+QD)$, $T_{Smrs(12)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}D_2+QD}(\bar{X}D_2+QD)$,

$T_{Smrs(13)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}D_3+QD}(\bar{X}D_3+QD)$, $T_{Smrs(14)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}D_4+QD}(\bar{X}D_4+QD)$,

$T_{Smrs(15)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}D_5+QD}(\bar{X}D_5+QD)$, $T_{Smrs(16)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}D_6+QD}(\bar{X}D_6+QD)$,

$T_{Smrs(17)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}D_7+QD}(\bar{X}D_7+QD)$, $T_{Smrs(18)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}D_8+QD}(\bar{X}D_8+QD)$,

$T_{Smrs(19)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}D_9+QD}(\bar{X}D_9+QD)$, $T_{Smrs(20)} = \frac{\bar{y}+\hat{\beta}(\bar{X}-\bar{x})}{\bar{x}D_{10}+QD}(\bar{X}D_{10}+QD)$.

The MSEs of the estimators $T_{Smrs(i)}$ $(i=1,2,...,20)$, are given by
$MSE(T_{Smrs(i)}) = \left(\frac{1}{n}-\frac{1}{N}\right)\left(\alpha_i^2C_x^2+\left(1-\rho^2\right)C_y^2\right)\bar{Y}^2$, where $(i=1,2,...,20)$,
$\alpha_1 = \frac{QD\bar{X}}{QD\bar{X}+D_1}$, $\alpha_2 = \frac{QD\bar{X}}{QD\bar{X}+D_2}$, $\alpha_3 = \frac{QD\bar{X}}{QD\bar{X}+D_3}$, $\alpha_4 = \frac{QD\bar{X}}{QD\bar{X}+D_4}$, $\alpha_5 = \frac{QD\bar{X}}{QD\bar{X}+D_5}$, $\alpha_6 = \frac{QD\bar{X}}{QD\bar{X}+D_6}$, $\alpha_7 = \frac{QD\bar{X}}{QD\bar{X}+D_7}$, $\alpha_8 = \frac{QD\bar{X}}{QD\bar{X}+D_8}$, $\alpha_9 = \frac{QD\bar{X}}{QD\bar{X}+D_9}$, $\alpha_{10} = \frac{QD\bar{X}}{QD\bar{X}+D_{10}}$, $\alpha_{11} = \frac{D_1\bar{X}}{D_1\bar{X}+QD}$, $\alpha_{12} = \frac{D_2\bar{X}}{D_2\bar{X}+QD}$, $\alpha_{13} = \frac{D_3\bar{X}}{D_3\bar{X}+QD}$, $\alpha_{14} = \frac{D_4\bar{X}}{D_4\bar{X}+QD}$, $\alpha_{15} = \frac{D_5\bar{X}}{D_5\bar{X}+QD}$, $\alpha_{16} = \frac{D_6\bar{X}}{D_6\bar{X}+QD}$, $\alpha_{17} = \frac{D_7\bar{X}}{D_7\bar{X}+QD}$, $\alpha_{18} = \frac{D_8\bar{X}}{D_8\bar{X}+QD}$, $\alpha_{19} = \frac{D_9\bar{X}}{D_9\bar{X}+QD}$, $\alpha_{20} = \frac{D_{10}\bar{X}}{D_{10}\bar{X}+QD}$.

Motivated by the work of Subzar *et al.* (2017), we suggest some linear regression type ratio exponential estimators for estimating population mean of the study variable $\bar{Y}$ using quartile deviation and deciles of auxiliary variable in survey sampling. We have also formulated the condition which makes the suggested classes of estimators more efficient than others and have shown that under this condition they are really more efficient than the existing estimators on the basis of numerical results in this literature.

3. Mathematical Formulation of Suggested Classes of Linear Regression Type Ratio Exponential Estimators

We suggest two classes (Class A and B) of efficient linear regression type ratio exponential estimators for estimating the population mean $\bar{Y}$ using the known population values of quartile deviation and deciles of auxiliary variable.

3.1. The First Suggested Class of Linear Regression Type Ratio Exponential Estimators (Class A)

The first suggested estimators $T_{SP1(j)}(j = 1,2,...,10)$ considered the linear regression type ratio exponential estimators for estimating population mean of the study variable $\bar{Y}$ by using the linear combination of known population values of quartile deviation (QD) and deciles $(D_j(j = 1,2,...,10))$ of an auxiliary variable in survey sampling:

$$T_{SP1(j)} = \left[\bar{y} + \hat{\beta}\left(\bar{X} - \bar{x}\right)\right] \exp\left[\Phi_j \frac{1 - \frac{\bar{x}}{\bar{X}}}{1 + \frac{QD\bar{x}+D_j}{QD\bar{X}+D_j}}\right], \tag{1}$$

where $\hat{\beta} = \frac{s_{yx}}{s_x^2}$, $s_{yx} = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})(x_i - \bar{x})$, $s_x^2 = (n-1)^{-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$, and $\Phi_j = \frac{QD\bar{X}}{QD\bar{X}+D_j}$,(j = 1, 2, ..., 10).

3.2. The Second Suggested Class of Linear Regression Type Ratio Exponential Estimators (Class B)

The second suggested estimators $T_{SP2(j)}(j = 1,2,...,10)$ considered the linear regression type ratio exponential estimators for estimating population mean of the study variable $\bar{Y}$ by using the linear combination of known population values of deciles $(D_j(j = 1,2,...,10))$ and quartile deviation (QD) of an auxiliary variable in survey sampling:

$$T_{SP2(j)} = \left[\bar{y} + \hat{\beta}\left(\bar{X} - \bar{x}\right)\right] \exp\left[\Psi_j \frac{1 - \frac{\bar{x}}{\bar{X}}}{1 + \frac{D_j\bar{x}+QD}{D_j\bar{X}+QD}}\right], \tag{2}$$

where $\hat{\beta} = \frac{s_{yx}}{s_x^2}$, $s_{yx} = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})(x_i - \bar{x})$, $s_x^2 = (n-1)^{-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$, and $\Psi_j = \frac{D_j\bar{X}}{D_j\bar{X}+QD}$, (j = 1, 2, ..., 10).
We get some members of the family of suggested estimators $T_{SP1(j)}$ and $T_{SP2(j)}$ in Table 1.

4. Behaviours of the suggested estimators $T_{SP1(j)}$ and $T_{SP2(j)}$ $(j = 1,2,...,10)$

To obtain the bias and mean square error (MSE) of the suggested estimators $T_{SP1(j)}$ and $T_{SP2(j)}$, $(j = 1,2,...,10)$ up-to the first order of large sample approximations are derived under the following transformations:
$\bar{y} = \bar{Y}(1+e_0)$, $\bar{x} = \bar{X}(1+e_1)$, $s_{yx} = S_{yx}(1+e_2)$, and $s_x^2 = S_x^2(1+e_3)$ such that $E(e_j) =$

0, $|e_j| < 1 \forall\ j = 0,1,2,3.$, $E(e_0^2) = (\frac{1}{n} - \frac{1}{N})C_y^2$, $E(e_1^2) = (\frac{1}{n} - \frac{1}{N})C_x^2$, $E(e_2^2) = (\frac{1}{n} - \frac{1}{N})\frac{\mu_{22}}{S_{yx}^2}$, $E(e_0e_1) = (\frac{1}{n} - \frac{1}{N})\rho_{yx}C_yC_x$, $E(e_1e_2) = (\frac{1}{n} - \frac{1}{N})\frac{\mu_{12}}{\bar{X}S_{yx}}$, $E(e_1e_3) = (\frac{1}{n} - \frac{1}{N})\frac{\mu_{03}}{\bar{X}S_x^2}$,
where $\mu_{rs} = E[(y-\bar{Y})^r(x-\bar{X})^s]$, r and s being positive integers.
Under the above transformations, expressing the equations "(1) and (2)" in terms of e's, we have

$$\begin{aligned} T_{SP1(j)} &= \left[\bar{y} + \frac{s_{yx}}{s_x^2}(\bar{X}-\bar{x})\right]\exp\left[\Phi_j \frac{1-\frac{\bar{x}}{\bar{X}}}{1+\frac{QD\bar{x}+D_j}{QD\bar{X}+D_j}}\right], \\ &= \left\{\bar{Y}(1+e_0) - \bar{X}\beta e_1(1+e_2)(1+e_3)^{-1}\right\}\exp\left[-\frac{1}{2}\Phi_j e_1\left(1+\frac{1}{2}\Phi_j e_1\right)^{-1}\right], \end{aligned} \quad (3)$$

$$\begin{aligned} T_{SP2(j)} &= \left[\bar{y} + \frac{s_{yx}}{s_x^2}(\bar{X}-\bar{x})\right]\exp\left[\Psi_j \frac{1-\frac{\bar{x}}{\bar{X}}}{1+\frac{D_j\bar{x}+QD}{D_j\bar{X}+QD}}\right], \\ &= \left\{\bar{Y}(1+e_0) - \bar{X}\beta e_1(1+e_2)(1+e_3)^{-1}\right\}\exp\left[-\frac{1}{2}\Psi_j e_1\left(1+\frac{1}{2}\Psi_j e_1\right)^{-1}\right], \end{aligned} \quad (4)$$

where $\beta = \frac{S_{yx}}{S_x^2}$.
Expanding the right side of "(3) and (4)", multiplying and neglecting the terms of $e's$ having power greater than two, we get

$$T_{SP1(j)} \cong \bar{Y}\left[1 + e_0 - \frac{1}{2}\Phi_j e_1 + \frac{3}{8}\Phi_j^2 e_1^2 - \frac{1}{2}\Phi_j e_0 e_1 - \frac{\bar{X}\beta}{\bar{Y}}\left(e_1 - \frac{1}{2}\Phi_j e_1^2 + e_1e_2 - e_1e_3\right)\right]. \quad (5)$$

$$T_{SP2(j)} \cong \bar{Y}\left[1 + e_0 - \frac{1}{2}\Psi_j e_1 + \frac{3}{8}\Psi_j^2 e_1^2 - \frac{1}{2}\Psi_j e_0 e_1 - \frac{\bar{X}\beta}{\bar{Y}}\left(e_1 - \frac{1}{2}\Psi_j e_1^2 + e_1e_2 - e_1e_3\right)\right]. \quad (6)$$

or

$$T_{SP1(j)} - \bar{Y} \cong \bar{Y}\left[e_0 - \frac{1}{2}\Phi_j e_1 + \frac{3}{8}\Phi_j^2 e_1^2 - \frac{1}{2}\Phi_j e_0 e_1 - B\left(e_1 - \frac{1}{2}\Phi_j e_1^2 + e_1e_2 - e_1e_3\right)\right], \quad (7)$$

$$T_{SP2(j)} - \bar{Y} \cong \bar{Y}\left[e_0 - \frac{1}{2}\Psi_j e_1 + \frac{3}{8}\Psi_j^2 e_1^2 - \frac{1}{2}\Psi_j e_0 e_1 - B\left(e_1 - \frac{1}{2}\Psi_j e_1^2 + e_1e_2 - e_1e_3\right)\right], \quad (8)$$

where $B = \rho\frac{C_y}{C_x}$.
Taking expectation of both sides of equations "(7) and (8)", we get the biases of the sug-

gested estimators up-to first order of large approximations as

$$\begin{aligned}
Bias[T_{SP1(j)}] &= E\left[T_{SP1(j)}-\bar{Y}\right], \\
&= \bar{Y}E\left[e_0-\frac{1}{2}\Phi_j e_1+\frac{3}{8}\Phi_j^2 e_1^2-\frac{1}{2}\Phi_j e_0 e_1-B\left(e_1-\frac{1}{2}\Phi_j e_1^2+e_1 e_2-e_1 e_3\right)\right], \\
&= \left(\frac{1}{n}-\frac{1}{N}\right)\bar{Y}\left(\frac{3}{8}\Phi_j^2 C_x^2+\frac{B}{\bar{X}^2 C_x}\{\frac{\mu_{03}}{\bar{X}C_x}-\frac{\mu_{12}}{\bar{Y}\rho C_y}\}\right). \qquad (9)
\end{aligned}$$

$$\begin{aligned}
Bias[T_{SP2(j)}] &= E\left[T_{SP2(j)}-\bar{Y}\right], \\
&= \bar{Y}\left[e_0-\frac{1}{2}\Psi_j e_1+\frac{3}{8}\Psi_j^2 e_1^2-\frac{1}{2}\Psi_j e_0 e_1-B\left(e_1-\frac{1}{2}\Psi_j e_1^2+e_1 e_2-e_1 e_3\right)\right], \\
&= \left(\frac{1}{n}-\frac{1}{N}\right)\bar{Y}\left(\frac{3}{8}\Psi_j^2 C_x^2+\frac{B}{\bar{X}^2 C_x}\{\frac{\mu_{03}}{\bar{X}C_x}-\frac{\mu_{12}}{\bar{Y}\rho C_y}\}\right). \qquad (10)
\end{aligned}$$

Now, after squaring of both sides of equations "(7) and (8)" and neglecting the terms of $e's$ having power of more than two, we have

$$\left[T_{SP1(j)}-\bar{Y}\right]^2=\bar{Y}^2\left[e_0^2+e_1^2\left(\frac{1}{2}\Phi_j+B\right)^2-2e_0 e_1\left(\frac{1}{2}\Phi_j+B\right)\right]. \qquad (11)$$

$$\left[T_{SP2(j)}-\bar{Y}\right]^2=\bar{Y}^2\left[e_0^2+e_1^2\left(\frac{1}{2}\Psi_j+B\right)^2-2e_0 e_1\left(\frac{1}{2}\Psi_j+B\right)\right]. \qquad (12)$$

Taking expectation of both sides of equations "(11) and (12)", we get the MSEs of the suggested estimators $T_{SP1(j)}$ and $T_{SP1(j)}$ (where $j=1,2,...,10$) for the first order of large approximations as

$$\begin{aligned}
MSE[T_{SP1(j)}] &= E\left[T_{SP1(j)}-\bar{Y}\right]^2, \\
&= \bar{Y}^2 E\left[e_0^2+e_1^2\left(\frac{1}{2}\Phi_j+B\right)^2-2e_0 e_1\left(\frac{1}{2}\Phi_j+B\right)\right], \\
&= \left(\frac{1}{n}-\frac{1}{N}\right)\bar{Y}^2\left[\frac{1}{4}\Phi_j^2 C_x^2+\left(1-\rho^2\right)C_y^2\right]. \qquad (13)
\end{aligned}$$

$$\begin{aligned}
MSE[T_{SP2(j)}] &= E\left[T_{SP2(j)}-\bar{Y}\right]^2, \\
&= \bar{Y}^2 E\left[e_0^2+e_1^2\left(\frac{1}{2}\Psi_j+B\right)^2-2e_0 e_1\left(\frac{1}{2}\Psi_j+B\right)\right], \\
&= \left(\frac{1}{n}-\frac{1}{N}\right)\bar{Y}^2\left[\frac{1}{4}\Psi_j^2 C_x^2+\left(1-\rho^2\right)C_y^2\right]. \qquad (14)
\end{aligned}$$

5. Efficiency Comparisons

In this section, the efficiency conditions for suggested estimators $T_{SP1(j)}$ and $T_{SP2(j)}$ ($j = 1,2,...,10$) have been derived algebraically according to the usual mean estimator, usual ratio (Cochran (1977)) estimator, Kadilar and Cingi (2004, 2006) estimators and Subzar *et al.* (2017) estimators.

5.1. Comparison with usual mean estimator

(i) $Var(\bar{y}) - MSE(T_{SP1(j)}) = \left(\frac{1}{n} - \frac{1}{N}\right)\bar{Y}^2\left[\rho^2 C_y^2 - \frac{1}{4}\Phi_j^2 C_x^2\right] > 0$, if

$$\frac{\rho C_y}{C_x} > \frac{1}{2}\Phi_j, (j = 1,2,...,10). \tag{15}$$

(ii) $Var(\bar{y}) - MSE(T_{SP2(j)}) = \left(\frac{1}{n} - \frac{1}{N}\right)\bar{Y}^2\left[\rho^2 C_y^2 - \frac{1}{4}\Psi_j^2 C_x^2\right] > 0$, if

$$\frac{\rho C_y}{C_x} > \frac{1}{2}\Psi_j, (j = 1,2,...,10). \tag{16}$$

5.2. Comparison with usual ratio estimator

(i) $MSE(\bar{y}_{Ratio}) - MSE(T_{SP1(j)}) = \left(\frac{1}{n} - \frac{1}{N}\right)\bar{Y}^2\left[(\rho C_y - C_x)^2 - \frac{1}{4}\Phi_j^2 C_x^2\right] > 0$, if

$$(\frac{\rho C_y}{C_x} - 1) > \frac{1}{2}\Phi_j, (j = 1,2,...,10). \tag{17}$$

(ii) $MSE(\bar{y}_{Ratio}) - MSE(T_{SP2(j)}) = \left(\frac{1}{n} - \frac{1}{N}\right)\bar{Y}^2\left[(\rho C_y - C_x)^2 - \frac{1}{4}\Psi_j^2 C_x^2\right] > 0$, if

$$(\frac{\rho C_y}{C_x} - 1) > \frac{1}{2}\Psi_j, (j = 1,2,...,10). \tag{18}$$

5.3. Comparison with Kadilar and Cingi (2004, 2006) estimators

(i) $(MSE(T_{KC(i)})) - MSE(T_{SP1(j)}) = \left(\frac{1}{n} - \frac{1}{N}\right)\bar{Y}^2\left[KC_i^2 - \frac{1}{4}\Phi_j^2\right] > 0$, if

$$KC_i > \frac{1}{2}\Phi_j, ((i = 1,2,...,10), (j = 1,2,...,10)). \tag{19}$$

(ii) $MSE(T_{KC(i)}) - MSE(T_{SP2(j)}) = \left(\frac{1}{n} - \frac{1}{N}\right)\bar{Y}^2\left[KC_i^2 - \frac{1}{4}\Psi_j^2\right] > 0$, if

$$KC_i > \frac{1}{2}\Psi_j, ((i = 1,2,...,10), (j = 1,2,...,10)). \tag{20}$$

5.4. Comparison with Subzar *et al.* (2017) estimators

(i) $MSE(T_{Smrs(i)}) - MSE(T_{SP1(j)}) = \left(\frac{1}{n} - \frac{1}{N}\right) \bar{Y}^2 \left[\alpha_i^2 - \frac{1}{4}\Phi_j^2\right] > 0$, if

$$\alpha_i > \frac{1}{2}\Phi_j, ((i = 1,2,...,20), (j = 1,2,...,10)). \tag{21}$$

(ii) $MSE(T_{Smrs(i)}) - MSE(T_{SP2(j)}) = \left(\frac{1}{n} - \frac{1}{N}\right) \bar{Y}^2 \left[\alpha_i^2 - \frac{1}{4}\Psi_j^2\right] > 0$, if

$$\alpha_i > \frac{1}{2}\Psi_j, ((i = 1,2,...,20), (j = 1,2,...,10)). \tag{22}$$

From the equations [(15)-(22)], the suggested classes of estimators $T_{SP1(j)}$ and $T_{SP2(j)}$ (where $j = 1,2,...,10$) are more efficient than the usual mean estimator, usual ratio (Cochran (1977)) estimator, Kadilar and Cingi (2004, 2006) estimators and Subzar *et al.* (2017) estimators as long as the conditions (15), (16), (17), (18), (19), (20), (21) and (22) are satisfied.

6. Numerical Demonstration

In this section, the suggested estimators are compared with respect to the some other existing estimators in this literature. The relative efficiencies $(\%)$ of the suggested estimators $T_{SP1(j)}$ and $T_{SP2(j)}$ (where $j = 1,2,...,10$) with respect to the usual mean estimator, usual ratio (Cochran (1977)) estimator, Kadilar and Cingi (2004, 2006) estimators and Subzar *et al.* (2017) estimators respectively, are computed as follows:

$$RE(ExistingEstimators, SuggestedEstimators) = \frac{MSE(ExistingEstimators)}{MSE(SuggestedEstimators)} \times 100.$$

The values of relative efficiencies $(\%)$ of the suggested estimators are shown in Tables [3-8]. Two different types of natural population data sets from the books (Murty (1967), page 228) and (Singh and Chaudhary (1986), page 177) have been considered to analyse the performance of the suggested estimators over other existing estimators.

7. Conclusions

In this paper, two natural population data sets have been considered for different parameters in Table 2. From Tables [3-8], it is found that our suggested classes of estimators are more efficient than the usual mean estimator, usual ratio (Cochran (1977)) estimator, Kadilar and Cingi (2004, 2006) estimators and Subzar *et al.* (2017) estimators. Hence, the performances of the suggested classes of linear regression type ratio exponential estimators are highly justified in numerical demonstration which are shown in Tables [3-8] that may be recommended for further use.

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Table 1: Some members of the suggested class of linear regression ratio type exponential estimators of Class A ($T_{SP1(j)}(j=1,2,...,10)$) and Class B ($T_{SP2(j)}(j=1,2,...,10)$) respectively.

The First Suggested Estimators (Class A)	Φ_i
$T_{SP1(1)} = \left[\bar{y}+\hat{\beta}(\bar{X}-\bar{x})\right] \exp\left[\Phi_1 \frac{(1-\bar{x}/\bar{X})}{1+((QD\bar{x}+D_1)/(QD\bar{X}+D_1))}\right]$	$\Phi_1 = QD\bar{X}/(QD\bar{X}+D_1)$
$T_{SP1(2)} = \left[\bar{y}+\hat{\beta}(\bar{X}-\bar{x})\right] \exp\left[\Phi_2 \frac{(1-\bar{x}/\bar{X})}{1+((QD\bar{x}+D_2)/(QD\bar{X}+D_2))}\right]$	$\Phi_2 = QD\bar{X}/(QD\bar{X}+D_2)$
$T_{SP1(3)} = \left[\bar{y}+\hat{\beta}(\bar{X}-\bar{x})\right] \exp\left[\Phi_3 \frac{(1-\bar{x}/\bar{X})}{1+((QD\bar{x}+D_3)/(QD\bar{X}+D_3))}\right]$	$\Phi_3 = QD\bar{X}/(QD\bar{X}+D_3)$
$T_{SP1(4)} = \left[\bar{y}+\hat{\beta}(\bar{X}-\bar{x})\right] \exp\left[\Phi_4 \frac{(1-\bar{x}/\bar{X})}{1+((QD\bar{x}+D_4)/(QD\bar{X}+D_4))}\right]$	$\Phi_4 = QD\bar{X}/(QD\bar{X}+D_4)$
$T_{SP1(5)} = \left[\bar{y}+\hat{\beta}(\bar{X}-\bar{x})\right] \exp\left[\Phi_5 \frac{(1-\bar{x}/\bar{X})}{1+((QD\bar{x}+D_5)/(QD\bar{X}+D_5))}\right]$	$\Phi_5 = QD\bar{X}/(QD\bar{X}+D_5)$
$T_{SP1(6)} = \left[\bar{y}+\hat{\beta}(\bar{X}-\bar{x})\right] \exp\left[\Phi_6 \frac{(1-\bar{x}/\bar{X})}{1+((QD\bar{x}+D_6)/(QD\bar{X}+D_6))}\right]$	$\Phi_6 = QD\bar{X}/(QD\bar{X}+D_6)$
$T_{SP1(7)} = \left[\bar{y}+\hat{\beta}(\bar{X}-\bar{x})\right] \exp\left[\Phi_7 \frac{(1-\bar{x}/\bar{X})}{1+((QD\bar{x}+D_7)/(QD\bar{X}+D_7))}\right]$	$\Phi_7 = QD\bar{X}/(QD\bar{X}+D_7)$
$T_{SP1(8)} = \left[\bar{y}+\hat{\beta}(\bar{X}-\bar{x})\right] \exp\left[\Phi_8 \frac{(1-\bar{x}/\bar{X})}{1+((QD\bar{x}+D_8)/(QD\bar{X}+D_8))}\right]$	$\Phi_8 = QD\bar{X}/(QD\bar{X}+D_8)$
$T_{SP1(9)} = \left[\bar{y}+\hat{\beta}(\bar{X}-\bar{x})\right] \exp\left[\Phi_9 \frac{(1-\bar{x}/\bar{X})}{1+((QD\bar{x}+D_9)/(QD\bar{X}+D_9))}\right]$	$\Phi_9 = QD\bar{X}/(QD\bar{X}+D_9)$
$T_{SP1(10)} = \left[\bar{y}+\hat{\beta}(\bar{X}-\bar{x})\right] \exp\left[\Phi_{10} \frac{(1-\bar{x}/\bar{X})}{1+((QD\bar{x}+D_{10})/(QD\bar{X}+D_{10}))}\right]$	$\Phi_{10} = QD\bar{X}/(QD\bar{X}+D_{10})$
The Second Suggested Estimators (Class B)	Ψ_i
$T_{SP2(1)} = \left[\bar{y}+\hat{\beta}(\bar{X}-\bar{x})\right] \exp\left[\Psi_1 \frac{(1-\bar{x}/\bar{X})}{1+((D_1\bar{x}+QD)/(D_1\bar{X}+QD))}\right]$	$\Psi_1 = D_1\bar{X}/(D_1\bar{X}+QD)$
$T_{SP2(2)} = \left[\bar{y}+\hat{\beta}(\bar{X}-\bar{x})\right] \exp\left[\Psi_2 \frac{(1-\bar{x}/\bar{X})}{1+((D_2\bar{x}+QD)/(D_2\bar{X}+QD))}\right]$	$\Psi_2 = D_2\bar{X}/(D_2\bar{X}+QD)$
$T_{SP2(3)} = \left[\bar{y}+\hat{\beta}(\bar{X}-\bar{x})\right] \exp\left[\Psi_3 \frac{(1-\bar{x}/\bar{X})}{1+((D_3\bar{x}+QD)/(D_3\bar{X}+QD))}\right]$	$\Psi_3 = D_3\bar{X}/(D_3\bar{X}+QD)$
$T_{SP2(4)} = \left[\bar{y}+\hat{\beta}(\bar{X}-\bar{x})\right] \exp\left[\Psi_4 \frac{(1-\bar{x}/\bar{X})}{1+((D_4\bar{x}+QD)/(D_4\bar{X}+QD))}\right]$	$\Psi_4 = D_4\bar{X}/(D_4\bar{X}+QD)$
$T_{SP2(5)} = \left[\bar{y}+\hat{\beta}(\bar{X}-\bar{x})\right] \exp\left[\Psi_5 \frac{(1-\bar{x}/\bar{X})}{1+((D_5\bar{x}+QD)/(D_5\bar{X}+QD))}\right]$	$\Psi_5 = D_5\bar{X}/(D_5\bar{X}+QD)$
$T_{SP2(6)} = \left[\bar{y}+\hat{\beta}(\bar{X}-\bar{x})\right] \exp\left[\Psi_6 \frac{(1-\bar{x}/\bar{X})}{1+((D_6\bar{x}+QD)/(D_6\bar{X}+QD))}\right]$	$\Psi_6 = D_6\bar{X}/(D_6\bar{X}+QD)$
$T_{SP2(7)} = \left[\bar{y}+\hat{\beta}(\bar{X}-\bar{x})\right] \exp\left[\Psi_7 \frac{(1-\bar{x}/\bar{X})}{1+((D_7\bar{x}+QD)/(D_7\bar{X}+QD))}\right]$	$\Psi_7 = D_7\bar{X}/(D_7\bar{X}+QD)$
$T_{SP2(8)} = \left[\bar{y}+\hat{\beta}(\bar{X}-\bar{x})\right] \exp\left[\Psi_8 \frac{(1-\bar{x}/\bar{X})}{1+((D_8\bar{x}+QD)/(D_8\bar{X}+QD))}\right]$	$\Psi_8 = D_8\bar{X}/(D_8\bar{X}+QD)$
$T_{SP2(9)} = \left[\bar{y}+\hat{\beta}(\bar{X}-\bar{x})\right] \exp\left[\Psi_9 \frac{(1-\bar{x}/\bar{X})}{1+((D_9\bar{x}+QD)/(D_9\bar{X}+QD))}\right]$	$\Psi_9 = D_9\bar{X}/(D_9\bar{X}+QD)$
$T_{SP2(10)} = \left[\bar{y}+\hat{\beta}(\bar{X}-\bar{x})\right] \exp\left[\Psi_{10} \frac{(1-\bar{x}/\bar{X})}{1+((D_{10}\bar{x}+QD)/(D_{10}\bar{X}+QD))}\right]$	$\Psi_{10} = D_{10}\bar{X}/(D_{10}\bar{X}+QD)$

Table 2: Parameters of two natural population data sets

Population A Murthy (1967), page 228		Population B Singh and Chaudhary (1986), page 177	
$N = 80$	$n = 20$	$N = 34$	$n = 20$
$\bar{Y} = 5182.637$	$\bar{X} = 1126.463$	$\bar{Y} = 856.4117$	$\bar{X} = 199.4412$
$\rho = 0.941$	$S_y = 1835.659$	$\rho = 0.4453$	$S_y = 733.1407$
$C_y = 0.354$	$S_x = 845.610$	$C_y = 0.8561$	$S_x = 150.2150$
$C_x = 0.751$	$\beta_2(x) = -0.063$	$C_x = 0.7531$	$\beta_2(x) = 1.0445$
$\beta_1(x) = 1.050$	$D_1 = 360$	$\beta_1(x) = 1.1823$	$D_1 = 60.60$
$D_2 = 460$	$D_3 = 590$	$D_2 = 83.00$	$D_3 = 102.70$
$D_4 = 670$	$D_5 = 750$	$D_4 = 111.20$	$D_5 = 142.50$
$D_6 = 850$	$D_7 = 1480$	$D_6 = 210.20$	$D_7 = 264.50$
$D_8 = 1810$	$D_9 = 2500$	$D_8 = 304.40$	$D_9 = 373.20$
$D_{10} = 3480$	$QD = 588.125$	$D_{10} = 634.00$	$QD = 80.25$

Table 3: Relative efficiencies (%) of the suggested estimators $T_{SP1(j)}$ and $T_{SP2(j)}$ $(j = 1,2,...,10)$ over the existing estimators $T_{KC(1)}$, $T_{KC(2)}$, $T_{KC(3)}$, $T_{KC(4)}$, $T_{KC(5)}$, $T_{KC(6)}$, $T_{KC(7)}$, $T_{KC(8)}$, $T_{KC(9)}$ and $T_{KC(10)}$ respectively for Population A.

$Estimators$	$T_{KC(1)}$	$T_{KC(2)}$	$T_{KC(3)}$	$T_{KC(4)}$	$T_{KC(5)}$	$T_{KC(6)}$	$T_{KC(7)}$	$T_{KC(8)}$	$T_{KC(9)}$	$T_{KC(10)}$
The First Suggested Estimators (Class A)										
$T_{SP1(1)}$	372.654	372.170	372.694	380.469	372.708	372.047	371.847	372.139	382.486	372.697
$T_{SP1(2)}$	372.756	372.272	372.797	380.573	372.810	372.149	371.949	372.241	382.591	372.799
$T_{SP1(3)}$	372.889	372.404	372.929	380.709	372.943	372.282	372.081	372.374	382.727	372.932
$T_{SP1(4)}$	372.970	372.486	373.011	380.792	373.024	372.363	372.162	372.455	382.811	373.013
$T_{SP1(5)}$	373.052	372.567	373.093	380.875	373.106	372.445	372.244	372.537	382.895	373.095
$T_{SP1(6)}$	373.154	372.669	373.195	380.980	373.208	372.547	372.346	372.639	383.000	373.197
$T_{SP1(7)}$	373.797	373.312	373.838	381.637	373.852	373.189	372.988	373.281	383.660	373.841
$T_{SP1(8)}$	374.134	373.648	374.175	381.981	374.189	373.526	373.324	373.618	384.006	374.178
$T_{SP1(9)}$	374.840	374.353	374.881	382.701	374.894	374.230	374.028	374.322	384.730	374.883
$T_{SP1(10)}$	375.842	375.354	375.883	383.724	375.897	375.231	375.028	375.323	385.759	375.886
The Second Suggested Estimators (Class B)										
$T_{SP2(1)}$	373.267	372.782	373.308	381.095	373.321	372.660	372.459	372.752	383.116	373.310
$T_{SP2(2)}$	373.054	372.569	373.095	380.877	373.108	372.447	372.246	372.539	382.897	373.097
$T_{SP2(3)}$	372.885	372.400	372.925	380.705	372.939	372.278	372.077	372.370	382.723	372.928
$T_{SP2(4)}$	372.813	372.329	372.854	380.632	372.867	372.207	372.006	372.299	382.650	372.857
$T_{SP2(5)}$	372.757	372.273	372.798	380.575	372.811	372.151	371.950	372.243	382.592	372.800
$T_{SP2(6)}$	372.702	372.218	372.742	380.518	372.756	372.095	371.895	372.187	382.535	372.745
$T_{SP2(7)}$	372.525	372.041	372.566	380.338	372.579	371.919	371.718	372.011	382.354	372.568
$T_{SP2(8)}$	372.482	371.998	372.522	380.293	372.536	371.875	371.675	371.967	382.309	372.525
$T_{SP2(9)}$	372.428	371.944	372.468	380.238	372.482	371.822	371.621	371.914	382.254	372.471
$T_{SP2(10)}$	372.388	371.904	372.429	380.198	372.442	371.782	371.581	371.874	382.213	372.431

Table 4: Relative efficiencies (%) of the suggested estimators $T_{SP1(j)}$ and $T_{SP2(j)}$ $(j = 1,2,...,10)$ over the existing estimators $T_{Smrs(i)}$ $(i = 1,2,...,10)$ respectively for Population A.

$Estimators$	$T_{Smrs(1)}$	$T_{Smrs(2)}$	$T_{Smrs(3)}$	$T_{Smrs(4)}$	$T_{Smrs(5)}$	$T_{Smrs(6)}$	$T_{Smrs(7)}$	$T_{Smrs(8)}$	$T_{Smrs(9)}$	$T_{Smrs(10)}$
The First Suggested Estimators (Class A)										
$T_{SP1(1)}$	372.259	372.150	372.007	371.920	371.832	371.723	371.036	370.676	369.927	368.866
$T_{SP1(2)}$	372.361	372.252	372.109	372.022	371.934	371.825	371.137	370.778	370.028	368.967
$T_{SP1(3)}$	372.494	372.384	372.242	372.154	372.067	371.957	371.269	370.910	370.160	369.098
$T_{SP1(4)}$	372.575	372.466	372.323	372.236	372.148	372.039	371.351	370.991	370.241	369.179
$T_{SP1(5)}$	372.657	372.547	372.405	372.317	372.230	372.120	371.432	371.072	370.322	369.26
$T_{SP1(6)}$	372.759	372.649	372.507	372.419	372.331	372.222	371.534	371.174	370.423	369.361
$T_{SP1(7)}$	373.401	373.292	373.149	373.061	372.973	372.864	372.174	371.814	371.062	369.998
$T_{SP1(8)}$	373.738	373.628	373.485	373.398	373.310	373.200	372.510	372.149	371.396	370.331
$T_{SP1(9)}$	374.443	374.333	374.190	374.102	374.014	373.904	373.212	372.851	372.097	371.030
$T_{SP1(10)}$	375.444	375.334	375.190	375.102	375.014	374.904	374.210	373.848	373.092	372.022
The Second Suggested Estimators (Class B)										
$T_{SP2(1)}$	372.872	372.762	372.620	372.532	372.444	372.335	371.646	371.286	370.535	369.473
$T_{SP2(2)}$	372.659	372.549	372.407	372.319	372.232	372.122	371.434	371.074	370.324	369.262
$T_{SP2(3)}$	372.490	372.380	372.238	372.150	372.063	371.953	371.265	370.906	370.156	369.094
$T_{SP2(4)}$	372.419	372.309	372.167	372.079	371.992	371.882	371.194	370.835	370.085	369.024
$T_{SP2(5)}$	372.362	372.253	372.111	372.023	371.935	371.826	371.138	370.779	370.029	368.968
$T_{SP2(6)}$	372.307	372.198	372.055	371.968	371.880	371.771	371.083	370.724	369.974	368.913
$T_{SP2(7)}$	372.130	372.021	371.879	371.791	371.704	371.595	370.907	370.548	369.799	368.738
$T_{SP2(8)}$	372.087	371.978	371.835	371.748	371.660	371.551	370.864	370.505	369.756	368.695
$T_{SP2(9)}$	372.033	371.924	371.782	371.694	371.607	371.498	370.810	370.451	369.702	368.642
$T_{SP2(10)}$	371.994	371.884	371.742	371.655	371.567	371.458	370.771	370.412	369.663	368.603

Table 5: Relative efficiencies $(\%)$ of the suggested estimators $T_{SP1(j)}$ and $T_{SP2(j)}$ $(j = 1,2,...,10)$ over the existing estimators $T_{Smrs(i)}$ $(i = 11,12,...,20)$ respectively for Population A.

$Estimators$	$T_{Smrs(11)}$	$T_{Smrs(12)}$	$T_{Smrs(13)}$	$T_{Smrs(14)}$	$T_{Smrs(15)}$	$T_{Smrs(16)}$	$T_{Smrs(17)}$	$T_{Smrs(18)}$	$T_{Smrs(19)}$	$T_{Smrs(20)}$
The First Suggested Estimators (Class A)										
$T_{SP1(1)}$	371.602	371.830	372.012	372.088	372.148	372.208	372.398	372.444	372.502	372.545
$T_{SP1(2)}$	371.704	371.932	372.113	372.190	372.250	372.310	372.500	372.546	372.604	372.647
$T_{SP1(3)}$	371.836	372.065	372.246	372.323	372.383	372.442	372.632	372.679	372.737	372.779
$T_{SP1(4)}$	371.918	372.146	372.327	372.404	372.464	372.524	372.714	372.760	372.818	372.861
$T_{SP1(5)}$	371.999	372.227	372.409	372.486	372.546	372.605	372.795	372.842	372.900	372.943
$T_{SP1(6)}$	372.101	372.329	372.511	372.587	372.648	372.707	372.897	372.944	373.002	373.045
$T_{SP1(7)}$	372.742	372.971	373.153	373.230	373.290	373.350	373.540	373.587	373.645	373.688
$T_{SP1(8)}$	373.078	373.308	373.490	373.567	373.627	373.687	373.877	373.924	373.982	374.025
$T_{SP1(9)}$	373.782	374.011	374.194	374.271	374.331	374.391	374.582	374.629	374.687	374.730
$T_{SP1(10)}$	374.781	375.012	375.194	375.272	375.332	375.392	375.584	375.631	375.689	375.732
The Second Suggested Estimators (Class B)										
$T_{SP2(1)}$	372.214	372.442	372.624	372.700	372.761	372.820	373.010	373.057	373.115	373.158
$T_{SP2(2)}$	372.001	372.229	372.411	372.488	372.548	372.607	372.797	372.844	372.902	372.945
$T_{SP2(3)}$	371.832	372.061	372.242	372.319	372.379	372.438	372.628	372.675	372.733	372.776
$T_{SP2(4)}$	371.761	371.989	372.171	372.247	372.308	372.367	372.557	372.604	372.661	372.704
$T_{SP2(5)}$	371.705	371.933	372.115	372.191	372.252	372.311	372.501	372.547	372.605	372.648
$T_{SP2(6)}$	371.650	371.878	372.059	372.136	372.196	372.256	372.445	372.492	372.550	372.593
$T_{SP2(7)}$	371.474	371.702	371.883	371.959	372.020	372.079	372.269	372.315	372.373	372.416
$T_{SP2(8)}$	371.430	371.658	371.839	371.916	371.976	372.036	372.225	372.272	372.330	372.373
$T_{SP2(9)}$	371.377	371.605	371.786	371.862	371.923	371.982	372.172	372.218	372.276	372.319
$T_{SP2(10)}$	371.337	371.565	371.746	371.823	371.883	371.942	372.132	372.179	372.236	372.279

Table 6: Relative efficiencies $(\%)$ of the suggested estimators $T_{SP1(j)}$ and $T_{SP2(j)}$ $(j = 1,2,...,10)$ over the existing estimators $T_{KC(1)}$, $T_{KC(2)}$, $T_{KC(3)}$, $T_{KC(4)}$, $T_{KC(5)}$, $T_{KC(6)}$, $T_{KC(7)}$, $T_{KC(8)}$, $T_{KC(9)}$ and $T_{KC(10)}$ respectively for Population B.

$Estimators$	$T_{KC(1)}$	$T_{KC(2)}$	$T_{KC(3)}$	$T_{KC(4)}$	$T_{KC(5)}$	$T_{KC(6)}$	$T_{KC(7)}$	$T_{KC(8)}$	$T_{KC(9)}$	$T_{KC(10)}$
The First Suggested Estimators (Class A)										
$T_{SP1(1)}$	158.552	157.968	157.743	157.992	157.481	158.206	158.093	157.248	158.221	156.753
$T_{SP1(2)}$	158.638	158.053	157.828	158.077	157.565	158.291	158.178	157.333	158.306	156.837
$T_{SP1(3)}$	158.713	158.127	157.902	158.152	157.640	158.366	158.252	157.407	158.380	156.911
$T_{SP1(4)}$	158.745	158.159	157.934	158.184	157.672	158.398	158.284	157.439	158.412	156.943
$T_{SP1(5)}$	158.863	158.277	158.052	158.302	157.789	158.516	158.402	157.556	158.531	157.060
$T_{SP1(6)}$	159.117	158.530	158.305	158.555	158.041	158.769	158.656	157.808	158.784	157.311
$T_{SP1(7)}$	159.319	158.732	158.506	158.756	158.242	158.971	158.857	158.009	158.986	157.511
$T_{SP1(8)}$	159.467	158.879	158.653	158.903	158.389	159.118	159.004	158.155	159.133	157.656
$T_{SP1(9)}$	159.719	159.130	158.904	159.155	158.639	159.370	159.256	158.406	159.385	157.906
$T_{SP1(10)}$	160.655	160.062	159.835	160.088	159.569	160.304	160.189	159.334	160.319	158.831
The Second Suggested Estimators (Class B)										
$T_{SP2(1)}$	158.726	158.141	157.916	158.165	157.653	158.379	158.266	157.421	158.394	156.924
$T_{SP2(2)}$	158.617	158.032	157.808	158.057	157.545	158.270	158.157	157.313	158.285	156.817
$T_{SP2(3)}$	158.560	157.976	157.751	158.000	157.489	158.214	158.101	157.256	158.229	156.760
$T_{SP2(4)}$	158.542	157.957	157.733	157.982	157.470	158.196	158.083	157.238	158.210	156.742
$T_{SP2(5)}$	158.494	157.909	157.685	157.934	157.422	158.147	158.034	157.190	158.162	156.694
$T_{SP2(6)}$	158.438	157.854	157.629	157.878	157.367	158.092	157.979	157.135	158.106	156.639
$T_{SP2(7)}$	158.414	157.830	157.605	157.854	157.343	158.068	157.955	157.111	158.082	156.615
$T_{SP2(8)}$	158.402	157.817	157.593	157.842	157.331	158.055	157.942	157.099	158.070	156.603
$T_{SP2(9)}$	158.387	157.802	157.578	157.827	157.316	158.040	157.927	157.084	158.055	156.589
$T_{SP2(10)}$	158.359	157.775	157.551	157.800	157.289	158.013	157.900	157.057	158.028	156.562

Table 7: Relative efficiencies (%) of the suggested estimators $T_{SP1(j)}$ and $T_{SP2(j)}$ $(j = 1,2,...,10)$ over the existing estimators $T_{Smrs(i)}$ $(i = 1,2,...,10)$ respectively for Population B.

$Estimators$	$T_{Smrs(1)}$	$T_{Smrs(2)}$	$T_{Smrs(3)}$	$T_{Smrs(4)}$	$T_{Smrs(5)}$	$T_{Smrs(6)}$	$T_{Smrs(7)}$	$T_{Smrs(8)}$	$T_{Smrs(9)}$	$T_{Smrs(10)}$
The First Suggested Estimators (Class A)										
$T_{SP1(1)}$	157.966	157.751	157.563	157.482	157.184	156.547	156.041	155.673	155.044	152.731
$T_{SP1(2)}$	158.051	157.836	157.647	157.566	157.269	156.631	156.125	155.756	155.127	152.813
$T_{SP1(3)}$	158.126	157.91	157.722	157.641	157.343	156.705	156.199	155.83	155.201	152.885
$T_{SP1(4)}$	158.158	157.942	157.754	157.673	157.375	156.736	156.23	155.862	155.232	152.916
$T_{SP1(5)}$	158.276	158.06	157.871	157.79	157.492	156.853	156.347	155.978	155.348	153.03
$T_{SP1(6)}$	158.529	158.313	158.124	158.042	157.744	157.104	156.597	156.227	155.596	153.275
$T_{SP1(7)}$	158.73	158.514	158.325	158.243	157.944	157.304	156.796	156.426	155.794	153.47
$T_{SP1(8)}$	158.877	158.661	158.471	158.39	158.09	157.449	156.941	156.57	155.938	153.612
$T_{SP1(9)}$	159.129	158.912	158.722	158.64	158.341	157.699	157.189	156.818	156.185	153.855
$T_{SP1(10)}$	160.061	159.843	159.652	159.57	159.269	158.623	158.11	157.737	157.1	154.756
The Second Suggested Estimators (Class B)										
$T_{SP2(1)}$	158.139	157.924	157.735	157.654	157.356	156.718	156.212	155.843	155.214	152.898
$T_{SP2(2)}$	158.031	157.815	157.627	157.546	157.248	156.61	156.105	155.736	155.107	152.793
$T_{SP2(3)}$	157.974	157.759	157.571	157.489	157.192	156.554	156.049	155.681	155.052	152.739
$T_{SP2(4)}$	157.956	157.741	157.552	157.471	157.174	156.536	156.031	155.663	155.034	152.721
$T_{SP2(5)}$	157.907	157.692	157.504	157.423	157.126	156.488	155.983	155.615	154.986	152.674
$T_{SP2(6)}$	157.852	157.637	157.449	157.368	157.071	156.434	155.928	155.56	154.932	152.621
$T_{SP2(7)}$	157.828	157.613	157.425	157.344	157.047	156.41	155.905	155.537	154.908	152.598
$T_{SP2(8)}$	157.816	157.601	157.413	157.332	157.035	156.398	155.893	155.525	154.896	152.586
$T_{SP2(9)}$	157.801	157.586	157.398	157.317	157.02	156.383	155.878	155.51	154.882	152.571
$T_{SP2(10)}$	157.774	157.559	157.371	157.29	156.993	156.356	155.851	155.483	154.855	152.545

Table 8: Relative efficiencies (%) of the suggested estimators $T_{SP1(j)}$ and $T_{SP2(j)}$ $(j = 1,2,...,10)$ over the existing estimators $T_{Smrs(i)}$ $(i = 11,12,...,20)$, respectively for Population B.

$Estimators$	$T_{Smrs(11)}$	$T_{Smrs(12)}$	$T_{Smrs(13)}$	$T_{Smrs(14)}$	$T_{Smrs(15)}$	$T_{Smrs(16)}$	$T_{Smrs(17)}$	$T_{Smrs(18)}$	$T_{Smrs(19)}$	$T_{Smrs(20)}$
The First Suggested Estimators (Class A)										
$T_{SP1(1)}$	157.529	157.803	157.946	157.992	158.115	158.255	158.316	158.347	158.385	158.454
$T_{SP1(2)}$	157.613	157.888	158.031	158.077	158.2	158.34	158.401	158.432	158.47	158.539
$T_{SP1(3)}$	157.688	157.962	158.105	158.151	158.274	158.415	158.476	158.507	158.545	158.614
$T_{SP1(4)}$	157.72	157.994	158.137	158.184	158.306	158.447	158.508	158.539	158.577	158.646
$T_{SP1(5)}$	157.837	158.112	158.255	158.301	158.424	158.565	158.626	158.657	158.695	158.764
$T_{SP1(6)}$	158.09	158.365	158.508	158.555	158.678	158.819	158.88	158.911	158.949	159.018
$T_{SP1(7)}$	158.29	158.566	158.71	158.756	158.879	159.02	159.082	159.113	159.151	159.220
$T_{SP1(8)}$	158.437	158.713	158.857	158.903	159.026	159.168	159.229	159.26	159.298	159.367
$T_{SP1(9)}$	158.688	158.964	159.108	159.155	159.278	159.42	159.481	159.512	159.55	159.620
$T_{SP1(10)}$	159.618	159.896	160.04	160.087	160.211	160.354	160.416	160.447	160.485	160.555
The Second Suggested Estimators (Class B)										
$T_{SP2(1)}$	157.701	157.976	158.119	158.165	158.288	158.428	158.489	158.52	158.558	158.627
$T_{SP2(2)}$	157.593	157.867	158.01	158.056	158.179	158.32	158.381	158.412	158.449	158.518
$T_{SP2(3)}$	157.536	157.811	157.954	158	158.123	158.263	158.324	158.355	158.393	158.462
$T_{SP2(4)}$	157.518	157.793	157.936	157.982	158.104	158.245	158.306	158.337	158.375	158.443
$T_{SP2(5)}$	157.47	157.744	157.887	157.933	158.056	158.196	158.257	158.288	158.326	158.395
$T_{SP2(6)}$	157.415	157.689	157.832	157.878	158	158.141	158.202	158.233	158.27	158.339
$T_{SP2(7)}$	157.391	157.665	157.808	157.854	157.976	158.117	158.178	158.209	158.246	158.315
$T_{SP2(8)}$	157.379	157.653	157.796	157.842	157.964	158.105	158.165	158.196	158.234	158.303
$T_{SP2(9)}$	157.364	157.638	157.781	157.827	157.949	158.09	158.151	158.181	158.219	158.288
$T_{SP2(10)}$	157.337	157.611	157.753	157.8	157.922	158.062	158.123	158.154	158.192	158.261