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## COORDINATION OF LOCOMOTIVES TURNOVER AND SERVICING MODES

**Summary.** A coordinated calculation of two processes – locomotives turnover and servicing – is described. Locomotives turnover is calculated by optimization system Labyrinth and servicing by dynamic transportation problem. Service programs integrate these processes. A model of coordinated arrival of locomotives at servicing stations is proposed. The calculation consists of three interrelated steps. The first step is the calculation of the optimal locomotive turnover without considering servicing constraints. Service program SP-1 determines stations where forced stops will take place according to the necessity of servicing and forms the basic location of locomotives for further movement to servicing stations. The second step is the calculation of the optimal arrival of locomotives at servicing stations. Service program SP-2 provides location and release time of each locomotive after servicing. The third step is the calculation of a train schedule with the consideration for the location of stopped train sets and the appearance of locomotives after servicing. Servicing program SP-3 forms the united results.

### 1. INTRODUCTION

An optimal locomotives turnover schedule is created by an optimization system Labyrinth. A formal statement of a problem, an approach to overcome the problem of huge size of resulting matrixes and a calculation method are described in articles [1] and [2]. A verification of the optimization system Labyrinth was carried out on a service area with the length of 1366 km (Fig. 1) for the different variants of locomotive fleet size (from 130 to 170 vehicles). Results of the verification are described in [3]. The articles also include results that completely characterize a pattern of locomotives utilization and a process of train flow movement in the given service area.

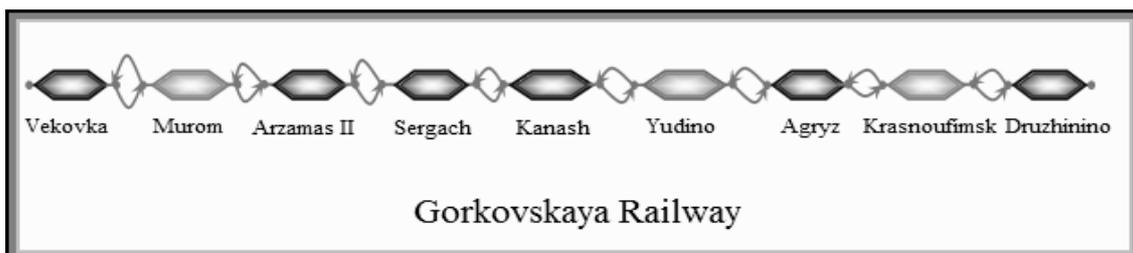


Fig. 1. Locomotives service area

It is also possible to get the information about each locomotive and each train (Tab. 1).

The optimization system Labyrinth is based on a dynamic transportation problem with delays (DTPD) [4]. In this problem, traffic flows connect points of production and consumption (otherwise – sources and drains). In DTPD, flows of train sets are considered as main flows; therefore, the sources and the drains are assigned for them. Flows of locomotives are connected to the flows of train sets with the corresponding constraints, and these two flows create train flows.

However, sources and drains are not assigned for locomotives because they are servicing means in this problem, but in order to represent the process of technical servicing, locomotives should become the main flow with sources and drains. Directly in the optimization system Labyrinth it is impossible to represent these two flows with the given connections between them. Therefore, it is necessary to carry out the calculation in several steps (Fig. 3).

Table 1

Locomotives utilization parameters

Full utilization	Useful utilization	Stabling	Idle time with a train	Empty run	Number of trains
[5] 20:56	[5] 17:36	02:24	03:20	00:00	6
[5] 20:40	[5] 17:12	01:04	03:28	00:00	5
[5] 19:20	[5] 16:00	03:12	03:20	00:00	5
[5] 19:52	[5] 15:44	01:44	04:08	00:00	7
[5] 18:48	[5] 15:04	03:20	03:44	00:00	5
[5] 18:08	[5] 14:32	04:16	03:36	00:00	5
[5] 17:04	[5] 14:32	06:00	02:32	00:00	8
[5] 17:44	[5] 14:32	02:40	03:12	00:00	7
[5] 19:04	[5] 14:24	02:56	04:40	00:00	5

The optimization system creates not only a train schedule but also a turnover schedule for each locomotive (Fig. 2).

DTPD-L model is a dynamic transportation problem with delays in a special statement where locomotives are considered as main flows.

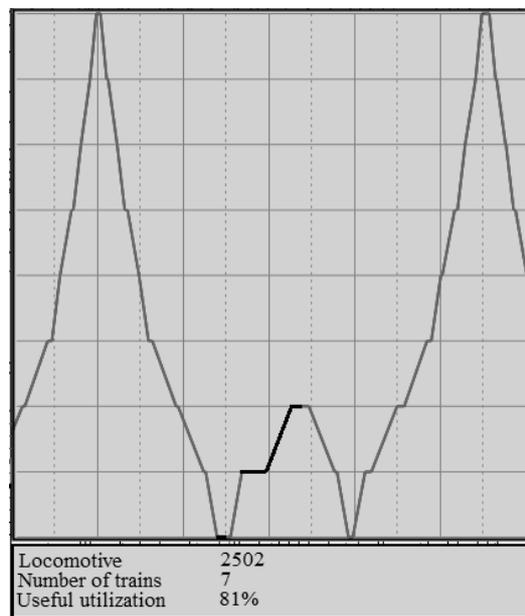
## 2. LITERATURE REVIEW

A large number of train locomotives optimization models have appeared in the literature since 1980s, but only some of these work, considering the servicing requirements of locomotives. Locomotive optimization models were studied by famous scientists such as Boole, Wright, Ziarati, Cordeau, Ahuja and Powell, and their works [5-14] made a huge contribution to the investigation of the locomotive scheduling problem. However, in the articles, they did not consider maintenance requirements of locomotives, and that is why, such models did not represent the real-world locomotive operations correctly. Maroti and Kroon were the first one who tried to overcome this problem [15]. They considered a problem of routing locomotives that are close to their servicing threshold and used multi-commodity flow model to solve the problem. Research group from the University of Florida continued to investigate this problem. In [16], they formulated the locomotive routing problem that considers refueling and maintenance. They solved the problem with aggregation/disaggregation

technique. The most recent work that considers servicing requirements of locomotives was carried out by a group of scientists from Princeton University. In [17], Powell et al. describe their application called Princeton Locomotive And Shop Management system (PLASMA) that consists of the following subsystems: PLASMA/SC, PLASMA/MC, and PLASMA/MA. PLASMA/MA captures such operational issues as routing locomotives to maintenance shops and refueling. In calculations, PLASMA uses approximate dynamic programming.

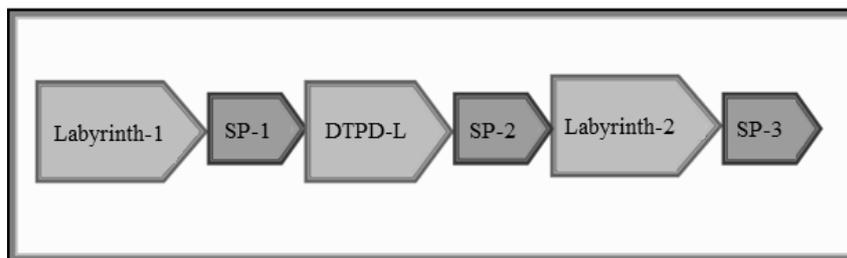
The necessity to create a domestic tool to calculate the optimal locomotive turnover schedule considering maintenance requirements was determined by the complexity of using foreign analogues connected with the impossibility to consider Russian features of technological process and information environment.

Our optimization tool is similar to American program PLASMA by the step principle of calculation. However, we propose a different calculation technique based on DTPD.



Black line shows idle time and empty running

Fig. 2. Locomotives turnover schedule



SP-1, SP-2, SP-3 – service programs

Fig. 3. Calculation plan of the optimal locomotives turnover with consideration for servicing constraints

### 3. CALCULATION STEPS OF AN OPTIMAL LOCOMOTIVES TURNOVER SCHEDULE WITH CONSIDERATION FOR SERVICING CONSTRAINTS

#### 3.1. Labyrinth-1

At this step, optimization system Labyrinth creates an optimal locomotive turnover schedule on a given period without considering servicing constraints.

### 3.2. Work of a service program SP-1

SP-1 executes the following actions:

1. Determination of locomotives limit work moments without servicing constraints.
2. Determination of stations where forced stops will take place using train schedule. All stations where forced stops take place will be technical because intermediate stations are not directly displayed.
3. Erasing of the train schedule after train stop moments.
4. Fixation of moments when the trains stop and their location. Stations where forced stops occur will be sources of train flows for Labyrinth-2.
5. Stations where trains stop become also the sources of locomotive flow that is used by DTPD-L program.
6. Formation of a calculation network for DTPD-L (Fig. 4). Stations where trains stop will be sources, servicing stations (technical stations with a locomotive depot) cannot be the drains, because it is impossible to set them the volumes of consumption (how many locomotives should every depot get serviced). That is why, the artificial drain (point  $z$ ) is introduced. It has the volume of consumption that is equal to the number of servicing locomotives and the time of consumption that is admittedly bigger than the time of a whole servicing process. Flows of locomotives on the way to the artificial drain necessarily visit servicing stations.
7. Formulation of the DTPD-L with the specific parameters (Fig. 4).

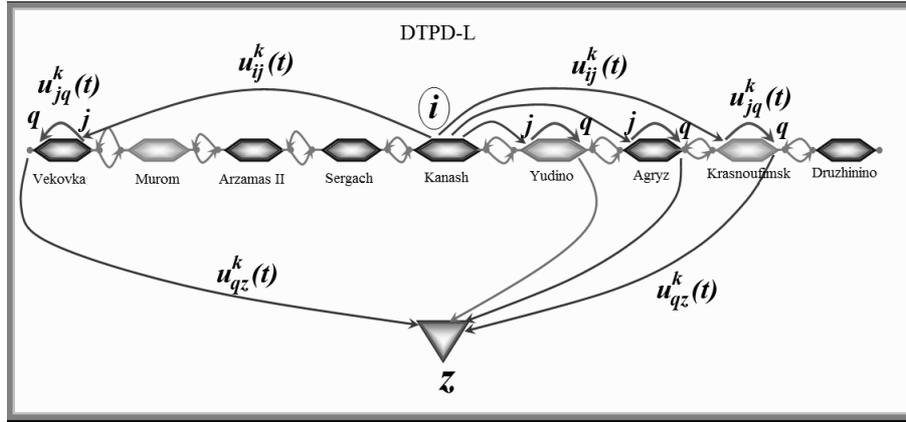


Fig. 4. Creation of the calculation network for the DTPD-L

Notations:  $u_{ij}^k(t)$  –  $k$  type locomotive flow from the place of location to the servicing station,

$u_{qz}^k(t)$  – flow towards the artificial drain  $z$ ,  $c_{qz}^k = 0$  – cost of locomotive flows movement to the artificial drain (for all types of locomotives).

Locomotive servicing scheme is shown in fig. 5. Locomotive either arrives at servicing facility, or waits for a vacant stabling place. To generate the problem, closed arcs to the artificial drain are introduced. They have zero cost.

8. Formation of the initial data for the specific calculation in DTPD-L program.

SP-1 forms the initial data for the DTPD-L. To do this, it is necessary to form a quantity of flows  $\{p_i^k(t_i^*)\}$  where  $p_i^k(t_i^*)$  is the presence of  $k$  type locomotives in  $i$  point at moment  $t_i^*$ . Moment  $t_i^*$  means time when a locomotive should visit a locomotive depot. Correspondingly a balance equation must be satisfied:

$$\forall i \left| p_i^k(t_i^*) = \sum_t u_{ij}^k(t) \right. \quad (1)$$

It means that all locomotives must be sent for servicing.

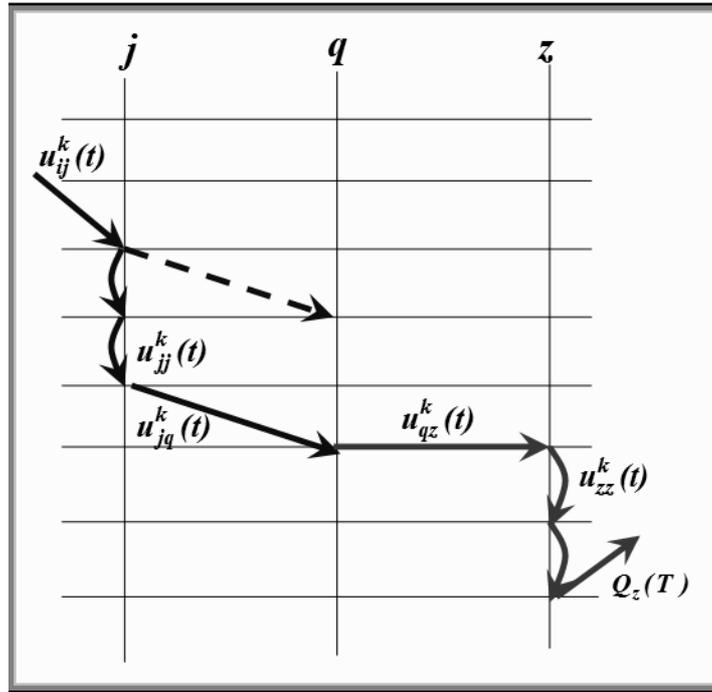


Fig. 5. Locomotive servicing scheme

### 3.3. Calculation of an optimal servicing process with the use of DTPD-L

Problem of the optimal servicing process calculation is set as a minimization of whole process costs

$$\sum_{(ij)} \sum_t \sum_k c_{ij}^k u_{ij}^k(t) + \sum_j \sum_t \sum_k c_{jj}^k u_{jj}^k(t) + \sum_j \sum_t \sum_k c_{jq}^k u_{jq}^k(t) \rightarrow \min \quad (2)$$

where:  $c_{jq}^k$  – cost of a  $k$ -type locomotive servicing at  $j$  point,  $0 \leq k \leq n$ ,  $n$  – number of locomotive types,  $c_{ij}^k(t)$  – costs connected with movement of one locomotive to the servicing station,  $u_{ij}^k(t)$  – locomotive flow from the place of location to the servicing station,  $c_{jj}^k(t)$  – cost of waiting for the servicing (for one locomotive),  $u_{jj}^k(t)$  – locomotives waiting for the servicing,  $u_{jq}^k(t)$  – locomotives under the process of servicing.

Meaning of the terms of the functional:

$$\sum_{(ij)} \sum_t \sum_k c_{ij}^k u_{ij}^k(t) \text{ – costs connected with movement of locomotives to the servicing stations. All}$$

the possible ways from stations of locomotive location to the servicing stations are taken into account. The calculation is carried out for each locomotive type separately.

$$\sum_j \sum_t \sum_k c_{jj}^k u_{jj}^k(t) \text{ – costs of waiting for the servicing.}$$

$$\sum_j \sum_t \sum_k c_{jq}^k u_{jq}^k(t) \text{ – costs of servicing.}$$

Artificial arcs ensure the correct statement of a problem and do not influence the results of a calculation because they have zero cost.

Constraints that ensure the technical servicing of all locomotives and take into account locomotive depots capacity must be satisfied while minimizing the functional:

1. Locomotive depot capacity constraint

$$u_{iq}^k(t) \leq d_{iq}^k \quad (3)$$

2. Constraint meaning that all locomotives must be serviced

$$\forall k \left| \sum_t \sum_q u_{jq}^k(t) \geq U_k \quad (4)$$

where:  $U_k$  – number of  $k$  type locomotives.

3. Coordination of sources and drains parameters

$$Q_z = \sum_q \sum_t \sum_k u_{jq}^k(t) \quad (5)$$

where:  $Q_z$  – volume of the artificial drain.

4. Balance equation of the locomotive depot

$$u_{jj}^k(t+1) = u_{jj}^k(t) + \sum_i u_{ij}^k(t - \tau_{ij}) - \sum_q u_{jq}^k(t) \quad (6)$$

### 3.4. Work of a service program SP-2

SP-2 transforms the output data of DTPD-L into the input data of the optimization system Labyrinth. Presence of locomotives in the  $j$ -point after the servicing will be the initial location for Labyrinth. Number of locomotives in each instant of time is determined by a variable  $u_{jq}^k(t)$ . Location of train sets is taken from the SP-1. Train schedule is created just as in optimization system Labyrinth.

### 3.5. Labyrinth-2

Calculation is carried out in dynamics with the new location of train sets and with the appearance of locomotives after servicing.

### 3.6. Work of a service program SP-3

SP-3 collects the results of Labyrinth-1 and Labyrinth-2 calculations, works them up, and forms the united array of train movement and locomotive turnover. Then, it calculates the parameters of this united process.

### 3.7. Calculation of an optimal mode of locomotives arrival at servicing stations

The problem of coordinated arrival of locomotives at servicing stations is very up-to-date. It is necessary to avoid queues at the locomotive depot. To make it possible, it is necessary to use a dynamic coordination method (DCM) [18].

DCM is based on DTPD, but it has variables correcting the output flows (Fig. 6).

Notations:  $\omega_i^k(t)$  – correcting variable,  $u_i^+(t)$  – the last train before stop,  $c_{ii}^k$  – cost of the locomotive idle time pending the departure to the servicing station,  $c_i^\omega$  – cost of correction.

Functional is the following:

$$\sum_{(ij)} \sum_t \sum_k c_{ij}^k u_{ij}^k(t) + \sum_i \sum_t \sum_k c_{ii}^k u_{ii}^k(t) + \sum_j \sum_t \sum_k c_{jq}^k u_{jq}^k(t) + \sum_i \sum_t c_i^\omega \omega_i(t) \rightarrow \min \quad (7)$$

In the functional costs of the locomotive, idle time at the departure stations and costs for the correction appear, but costs of the idle time at servicing stations are lacking. Using DCM, locomotives arrive in coordination just to the beginning of servicing process.

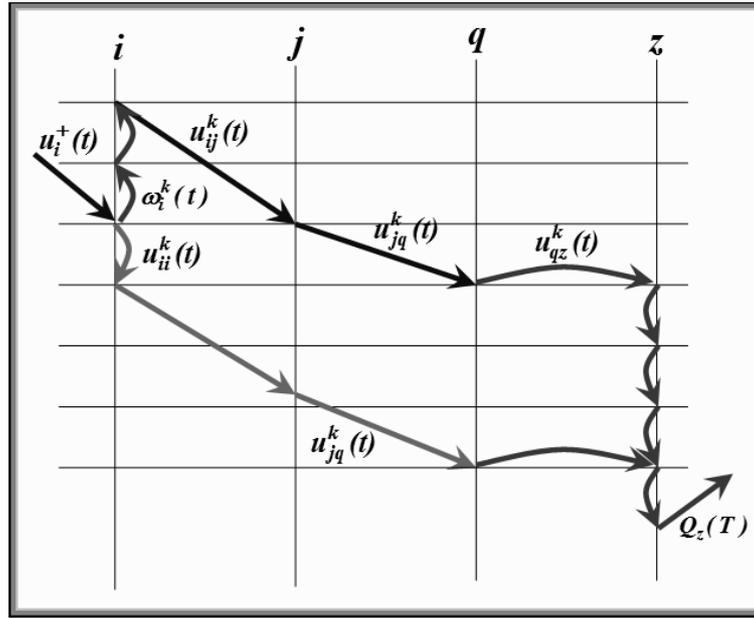


Fig. 6. Scheme of the coordinated arrival of locomotives at servicing stations

Balance equation at the departure station

$$u_{ii}^k(t+1) = u_{ii}^k(t) + u_i^+(t) - u_{ij}^k(t) + \omega_i(t+1) - \omega_i(t) \quad (8)$$

Initial constraints of the arcs constraints

$$u_{jq}(t) \leq d_{jq}; u_{ii}(t) \leq d_{ii}; u_{ij}(t) \leq d_{ij}; u_{jj}(t) \leq d_{jj} \quad (9)$$

Constraints that do not allow locomotives to stay at servicing stations

$$d_{jj} = 0; 0 \leq t \leq T \quad (10)$$

#### 4. CONCLUSION

Because of the impossibility to consider in Labyrinth the flows of trains and locomotives simultaneously, therefore to gain the optimal locomotive turnover schedule with consideration for servicing constraints, it was proposed to carry out the calculation in several steps. The optimal turnover schedule without servicing constraints is calculated in Labyrinth, and the specification of the results to organize locomotive servicing is carried out in DTPD-L. Coordination of the results obtained using these models is produced by service programs (SP-1, SP-2 and SP-3).

Moreover, the DCM is proposed. This method can be useful to estimate a rational location and an appearance rhythm of locomotives before servicing to organize their coordinated arrival at servicing stations.

Such step calculation system works automatically as a single program and can become an optimizing unit in the corresponding automatic control systems.

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